A particle in a simple harmonic oscillator potential energy well $V(x) = \frac{1}{2}kx^2$ has a wavefunction initialized in the second excited state:

$$\Psi(x, t = 0) = \psi_2(x)$$

Choose all of the statements that are correct.

A. The expectation value of the position of this particle depends on time *t*

B. The expectation value of the momentum of this particle depends on time *t*

C. The expectation value of the energy of this particle depends on time *t*

D. None of the above

CHM 305 The Quantum World

Lecture 11: Angular Momentum

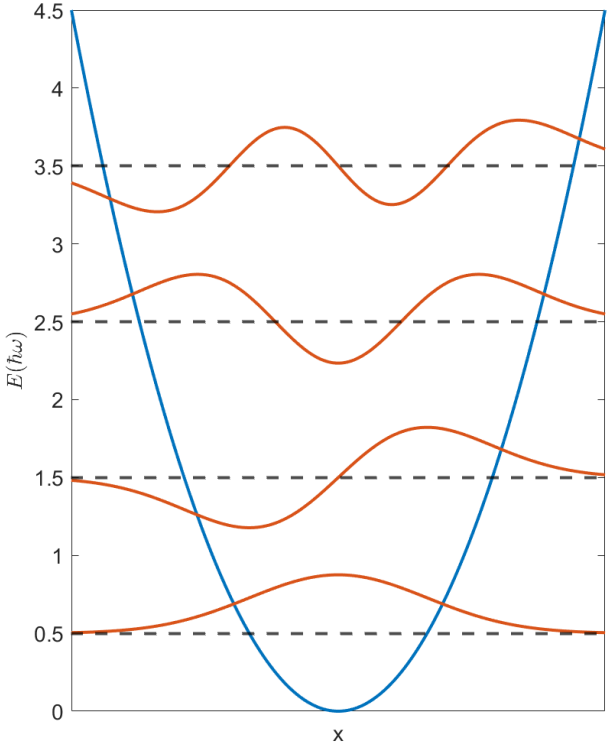
McQuarrie Ch. 6

Last time we discussed the quantum harmonic oscillator

- Briefly lay out what we know about the classical harmonic oscillator
- Introduce the quantum harmonic oscillator, and its relevance to molecular vibrations
- Write down the Schrödinger equation for this system, and examine the resulting wavefunctions and energy eigenvalues
- Learn about some nice properties of these wavefunctions

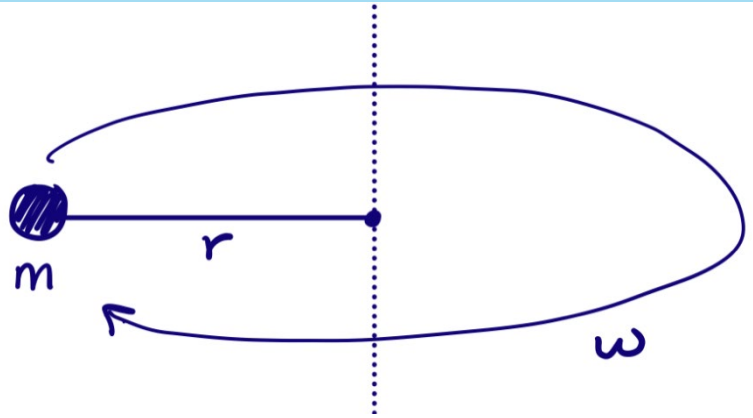


$$V(x) = \frac{1}{2}kx^2$$



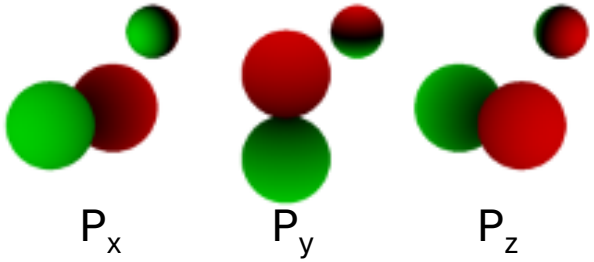
Road map for today: angular momentum

- Lay out definitions for classical circular motion and angular momentum
- Discuss rotations of classical and quantum mechanical rigid bodies
- Write down the Schrodinger equation and its solutions for the quantum rigid rotor in spherical coordinates
- Make connections to rotations of diatomic molecules and the hydrogen atom



$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2$$

$$\hat{H}\psi(\theta,\phi) = E\psi(\theta,\phi)$$



Practice Problem #1:

The rigid rotor Hamiltonian has the form

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\right] + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

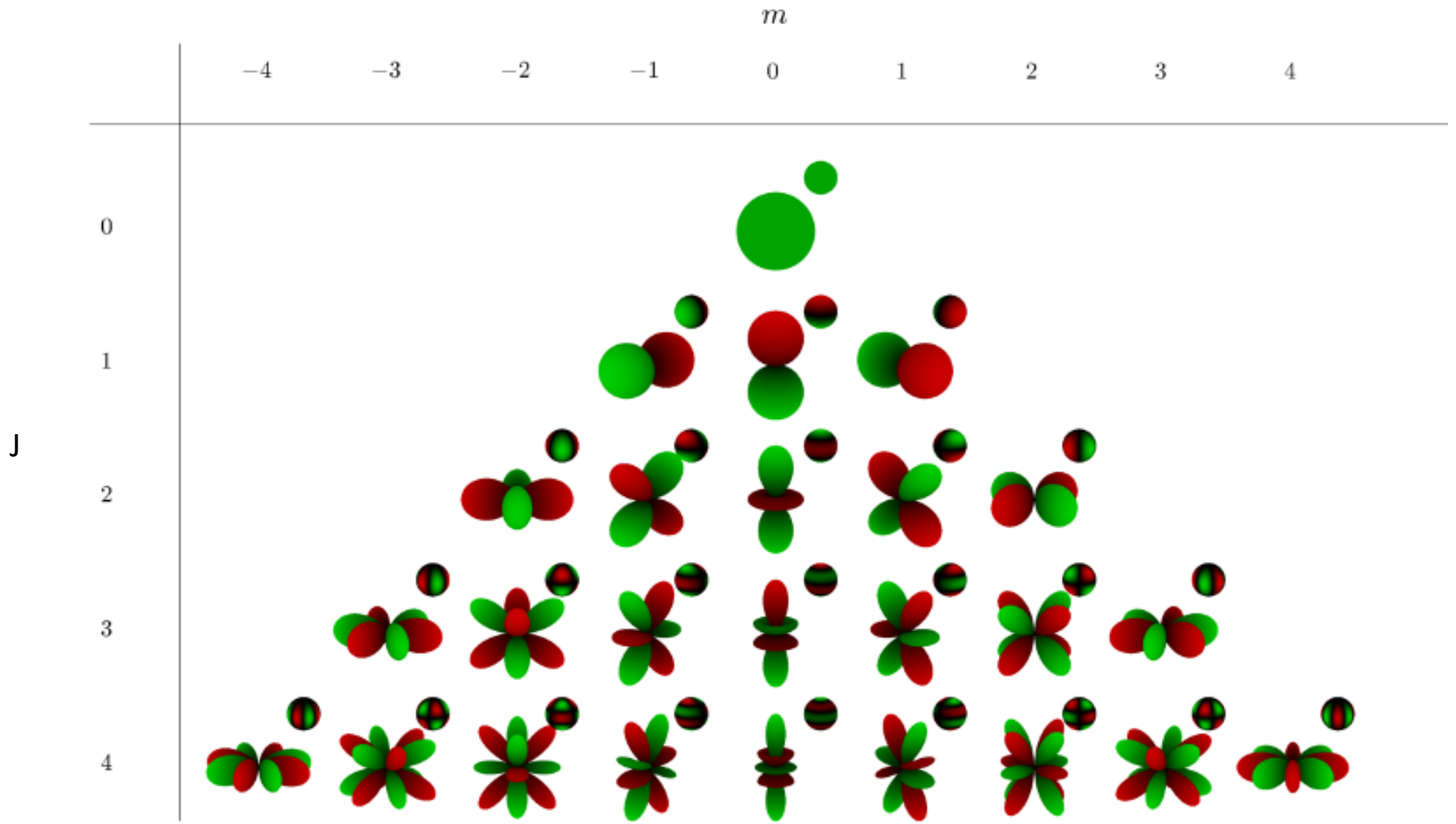
and can also be written as

$$\hat{H} = \frac{\hat{\ell}^2}{2I}$$

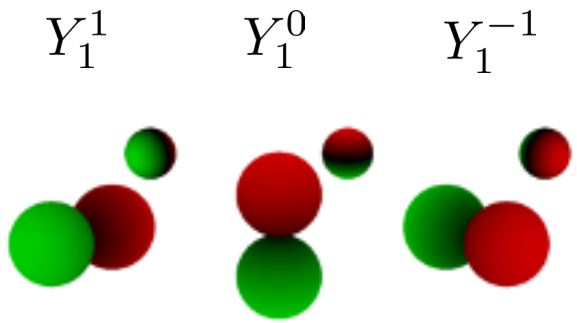
Do $\hat{H}$ and $\hat{\ell}^2$ commute? What does this tell you about their eigenfunctions?

Can you draw an analogy to the energy and linear momentum of a free particle?

Spherical harmonics



Practice Problem #2:



Show that

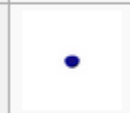
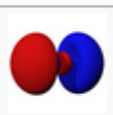
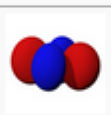
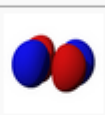
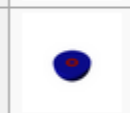
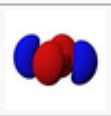
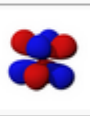
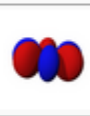
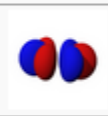
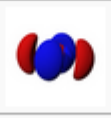
$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

is a solution to the rigid rotor Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\right] + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

and find its energy eigenvalue.

Hydrogen atom orbitals

	s ($\ell = 0$)	p ($\ell = 1$)			d ($\ell = 2$)					f ($\ell = 3$)						
	$m = 0$	$m = 0$	$m = \pm 1$		$m = 0$	$m = \pm 1$		$m = \pm 2$		$m = 0$	$m = \pm 1$		$m = \pm 2$		$m = \pm 3$	
	s	p_z	p_x	p_y	d_{z^2}	d_{xz}	d_{yz}	d_{xy}	$d_{x^2-y^2}$	f_{z^3}	f_{xz^2}	f_{yz^2}	f_{xyz}	$f_{z(x^2-y^2)}$	$f_{x(x^2-3y^2)}$	$f_{y(3x^2-y^2)}$
$n = 1$																
$n = 2$																
$n = 3$																
$n = 4$																
$n = 5$										...	...	...	...	...	...	...
$n = 6$					...	...	...	...	...	...	...	...	...	...	...	...
$n = 7$		...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

https://en.wikipedia.org/wiki/Atomic_orbital

Wrapping up

- **Next time:** Midterm review
- **Today's reading:** McQuarrie Ch. 6
- **In the last minute or two**:
  What would you most like to see reviewed before the midterm?