If the initial wavefunction for a 1D particle is $\Psi(x, t = 0) = \psi_2(x)$ (the first excited energy eigenstate), which of the following statements are correct? Select all that apply:

A. $\langle p \rangle = \int_{-\infty}^{\infty} \psi_2^*(x) \cdot -i\hbar \frac{\partial}{\partial x} \psi_2(x) dx$

B. $\langle p \rangle = \sum_{n=1}^{\infty} |c_n|^2 p_n$ where $\Psi(x, t = 0) = \sum_{n=1}^{\infty} c_n \phi_n(x)$ and $\phi_n(x)$ is the momentum eigenstate with momentum eigenvalue p_n

C. $\langle p \rangle = \int_{-\infty}^{\infty} -i\hbar \frac{\partial}{\partial x} |\psi_2(x)|^2 dx$

CHM 305 The Quantum World

Lecture 7: The Postulates of Quantum Mechanics (pt. 2)

Reading: McQuarrie Ch. 4

Some general test-taking advice

- **Familiarize yourself with the formula sheet in advance of the exam.** If something is given to you, you don't need to spend time deriving it.
- **Always show your work!** We may award partial credit for setting the problem up correctly, even if your final answer is incorrect.
- **Always label axes on plots.**
- **Perform a "common sense" sanity check** on your final answers: does the answer seem reasonable? Do the units work out correctly? (e.g. if you are evaluating a probability, is your answer between 0 and 1? If you are evaluating $\langle x \rangle$, does your answer have units of length?)

Last lecture we discussed…

The five *Postulates of Quantum Mechanics*

1. Wave functions represent probability distributions $P[x_1, x_2] = \int_{x_1}^{x_2} |\Psi(x,t)|^2 dx$
2. Operators represent physical quantities, or observables $x \rightarrow \hat{x}$, $E \rightarrow \hat{H}$
3. Measurements with operators read out eigenvalues $\hat{H} \sum_n c_n \psi_n(x) \rightarrow E_n$ with probability $|c_n|^2$
4. Expectation values of measurements $\langle E \rangle = \int_{-\infty}^{\infty} \psi^*(x) \hat{H} \psi(x) dx$
5. The time-dependent SE $\hat{H}\Psi(x,t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

Road map for today's lecture

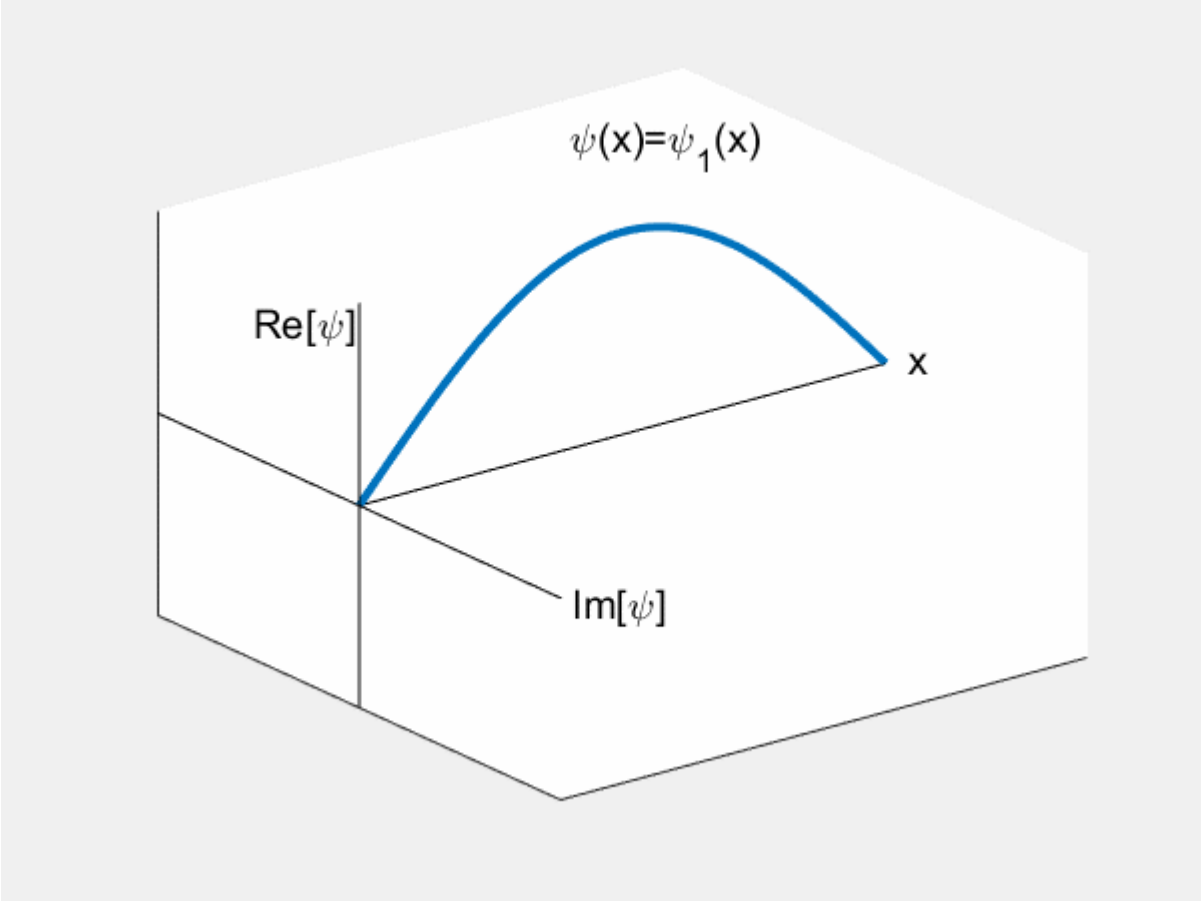
- **Postulate 5, the time-dependent Schrödinger equation.**
  Re-examine the workflow for solving for the time-dependence of $\Psi(x,t)$ given that the system was initialized in $\Psi(x,0)$ at time $t = 0$

- Examine the resulting time-dependent probability distributions $|\Psi(x,t)|^2$ for
  - Energy eigenstates
  - Superpositions of energy eigenstates

- Examine various time-dependent expectation values $\langle A \rangle$ for
  - Energy eigenstates
  - Superpositions of energy eigenstates

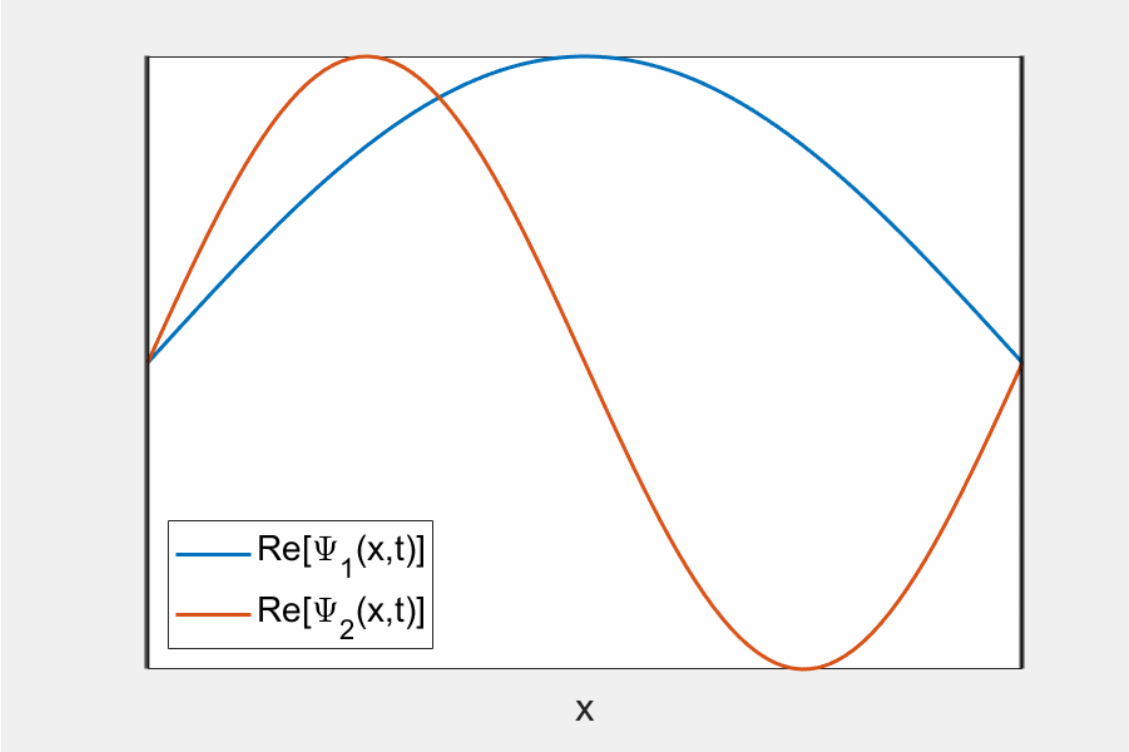
Plotting complex wavefunctions

imaginary and real parts of n=1 PIB eigenfunction



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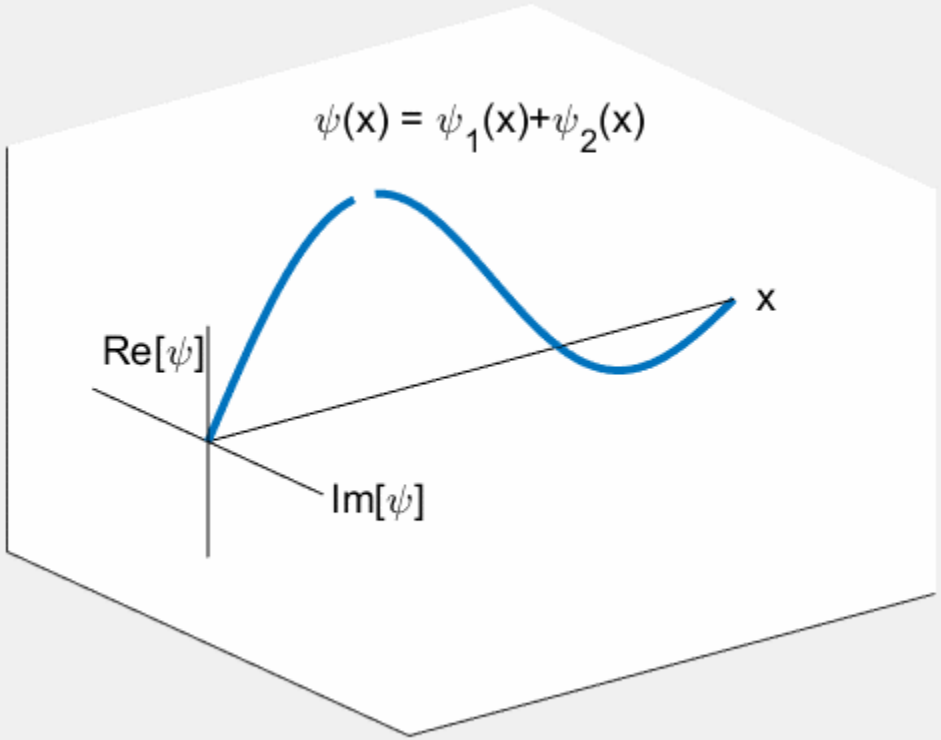
just the real parts of n=1, n=2 PIB eigenfunctions
(note n=2 state oscillates 4x faster than n=1)



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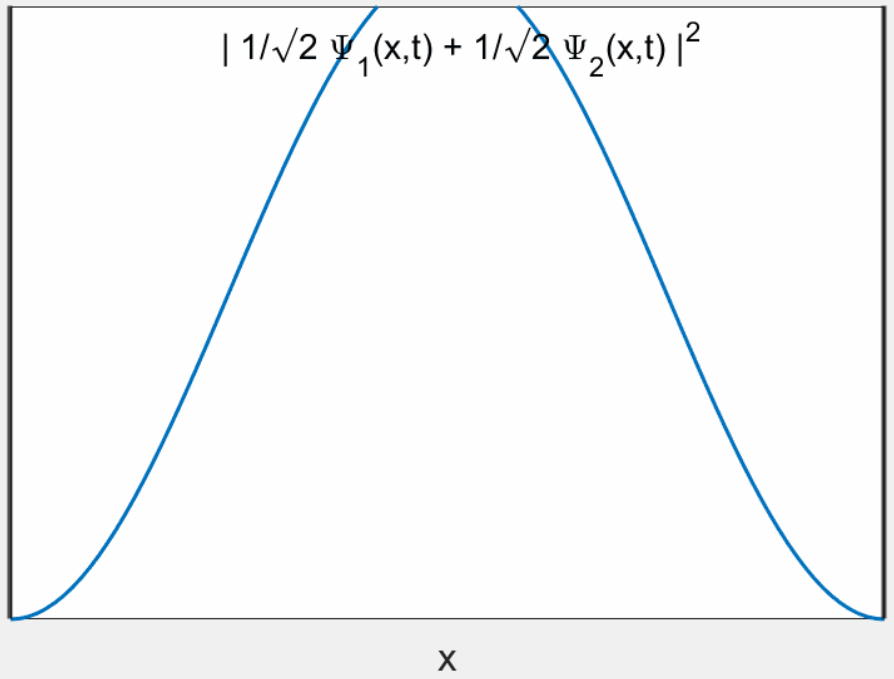
Probability distribution for a PIB superposition state

imaginary and real parts of for evenly weighted superposition of n=1, n=2 eigenfunctions



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probability distribution for evenly weighted superposition of n=1, n=2 eigenfunctions



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"sloshing" happens at the difference in frequency!

Practice problem #1

Consider a particle in a box of length a with wavefunction given by

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_1(x)e^{-i\omega_1 t} + \psi_2(x)e^{-i\omega_2 t}\right]$$

Consider the time-dependent probability that we find the particle in the left half of the box, $x = [0, a/2]$, by evaluating

$$P\left[0, \frac{a}{2}\right] = \int_0^{a/2} dx|\Psi(x,t)|^2$$

Show that this quantity takes the form (where c is some constant):

$$P\left[0, \frac{a}{2}\right] = \frac{1}{2} + c \cdot \cos((\omega_1 - \omega_2)t)$$

Practice problem #2

What is the expected value of the energy, $\langle E \rangle$, of the particle-in-a-box superposition state below?

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-i\omega_1 t} + \frac{1}{\sqrt{2}}\psi_2(x)e^{-i\omega_2 t}$$

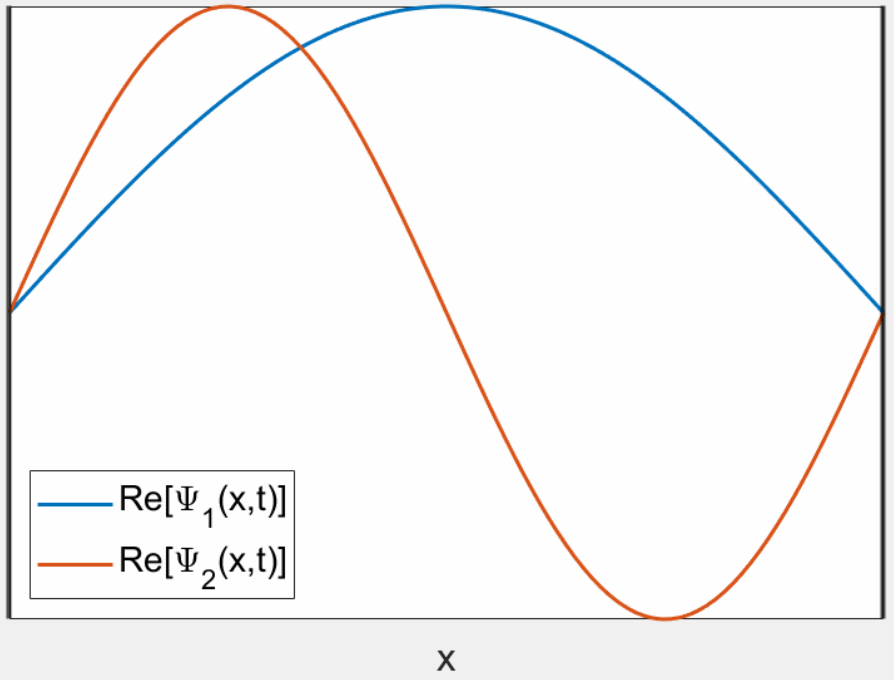
Practice problem #2.5

What can you say about the expectation value for the position, $\langle x \rangle$, of the same particle-in-a-box superposition state below?

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-i\omega_1 t} + \frac{1}{\sqrt{2}}\psi_2(x)e^{-i\omega_2 t}$$

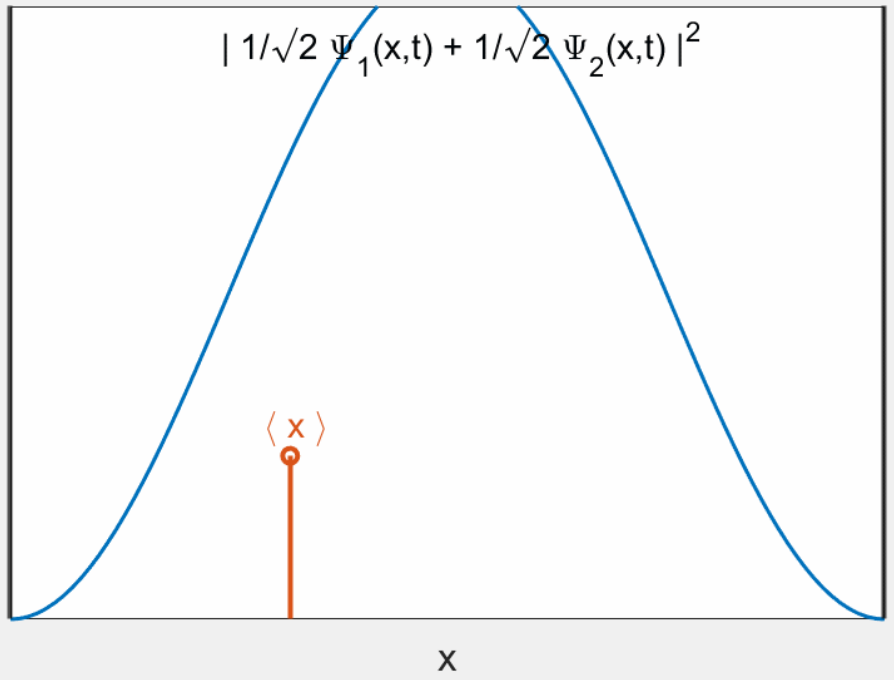
$\langle x \rangle$ for a PIB superposition state

just the real parts of n=1, n=2 PIB eigenfunctions (note n=2 state oscillates 4x faster than n=1)



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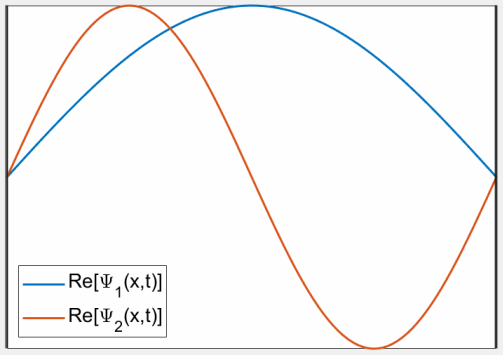
probability distribution for evenly weighted superposition of n=1, n=2 eigenfunctions



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In summary...

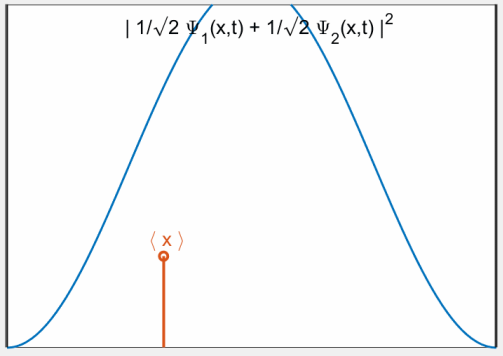
stationary states



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- $|\Psi(x,t)|^2$ is ***time-independent***
- $\langle E \rangle$ is ***time-independent***
- $\langle A \rangle$ is ***time-independent***

superposition states



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- $|\Psi(x,t)|^2$ is ***time-dependent***
- $\langle E \rangle$ is ***time-independent*** (magically!)
- $\langle A \rangle$ is ***time-dependent***
  (unless $\hat{A}$ commutes with the Hamiltonian)

Wrapping up

- **Next time:** quantum uncertainty
- **Today's reading:** McQuarrie Ch. 4
- **In the last minute or two**: One major takeaway from today's lecture?