Consider the temporal dynamics of $\Psi(x,t)$ when we initialize the system at time $t=0$ in the superposition state

$$\Psi(x,t=0) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + \psi_2(x)\right]$$

where $\psi_1(x)$ and $\psi_2(x)$ are two eigenstates of the 1D particle in a box. Which of the following statements are correct? **Select all that apply.**

(a) The probability distribution $|\Psi(x,t)|^2$ has time-dependent behavior

(b) The expectation value for the position of the particle $\langle x \rangle = \int_{-\infty}^{\infty} dx\ \Psi^*(x,t) \cdot \hat{x} \cdot \Psi(x,t)$ has time-dependent behavior

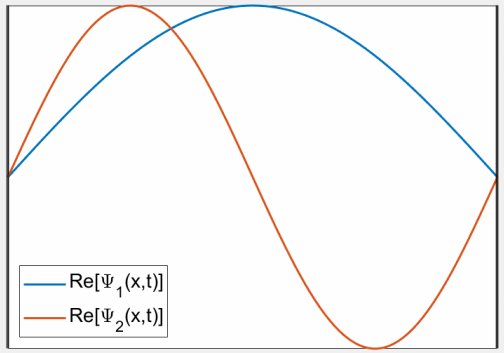
(c) The expectation value for the energy of the particle $\langle E \rangle = \int_{-\infty}^{\infty} dx\ \Psi^*(x,t) \cdot \hat{H} \cdot \Psi(x,t)$ has time-dependent behavior

CHM 305 The Quantum World

Lecture 8: The Uncertainty Principle

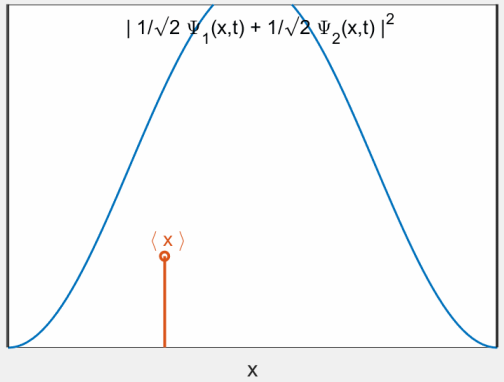
Last class we discussed…

stationary states



- probability distribution $|\Psi(x,t)|^2$ is ***time-independent***
- energy expectation value $\langle E \rangle$ is ***time-independent***
- arbitrary observable expectation value $\langle A \rangle$ is ***time-independent***

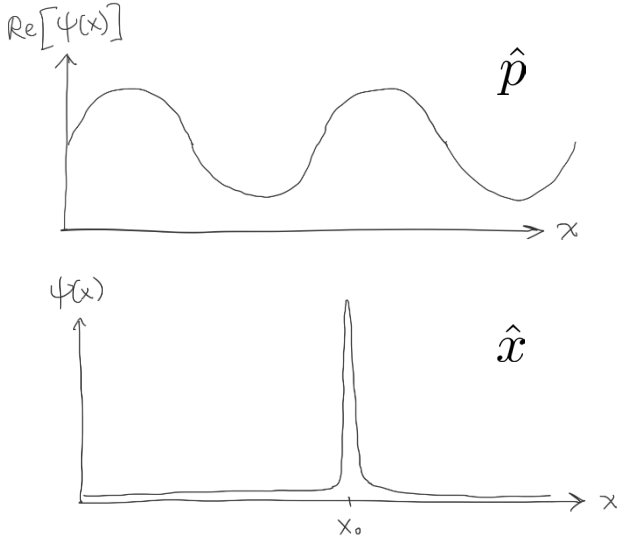
superposition states



- probability distribution $|\Psi(x,t)|^2$ is ***time-dependent***
- energy expectation value $\langle E \rangle$ is ***time-independent*** (magically!)
- arbitrary observable expectation value $\langle A \rangle$ is ***time-dependent*** (unless $\hat{A}$ commutes with the Hamiltonian)

Road map for today's lecture…

- Review the rules for quantum measurements and learn about "wavefunction collapse"
- Discuss trade-offs in uncertainty between knowing a free particle's (or a wave's) momentum and position
- Introduce the variance and standard deviation as metrics for the uncertainty of a measurement
- Introduce Heisenberg's uncertainty principle
- Discuss resulting consequences for operators that do and do not commute



$$\sigma_A^2 = \langle A^2 \rangle - \langle A \rangle^2$$

$$\sigma_A^2 \sigma_B^2 \geq -\frac{1}{4}\left(\int \psi^* \left[\hat{A}, \hat{B}\right] \psi \, dx \right)^2$$

Practice Problem #1: Uncertainty Principle

Apply the uncertainty principle

$$\sigma_A^2 \sigma_B^2 \geq -\frac{1}{4}\left(\int \psi^* \left[\hat{A}, \hat{B}\right] \psi \, dx \right)^2$$

to simultaneous measurements made with the $\hat{x} = x$ and $\hat{p} = -i\hbar\frac{d}{dx}$ quantum operators, and find an expression for $\sigma_x \sigma_p$.

Practice Problem #2: Commuting Operators

For a free particle, $V(x) = 0$ and therefore the Hamiltonian is just the kinetic energy operator

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

Does the momentum operator $\hat{p} = -i\hbar\frac{d}{dx}$ commute with this Hamiltonian?

What does this imply about the eigenfunctions of these two operators? Write down the eigenfunctions of each operator and confirm that they behave as you expect.

Wrapping up

- **Next time:** quantum tunneling
- **PS3** out today, due next Monday
- **Quiz #1 tonight! Tuesday 9/29,** Jadwin A08, 7:30 – 9:30 PM
- **In the last minute or two**: Muddiest point?