CHM 305 The Quantum World
Lecture 3: The Schrödinger equation

Thursday, September 9, 2021
Reading: McQuarrie Chap. 3
Quick review of what we learned last time...

- Show how classical waves have wavefunctions and are solutions to the classical wave equation

\[
\frac{\partial^2 \psi(x, t)}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 \psi(x, t)}{\partial t^2}
\]

- Solve the wave equation with separation of variables

- Show how the solutions to the wave equation with boundary conditions give us a set of standing wave solutions

- Classical standing waves will be a nice analogy for solutions to Schrödinger’s quantum wave equation

\[
\psi(x, t) = X(x) \cdot T(t)
\]
Standing wave normal modes

\[ n = 1 \quad \text{“first harmonic”} \quad n = 2 \quad \text{“second harmonic”} \quad n = 3 \quad \text{“third harmonic”} \quad n = 4 \quad \text{“fourth harmonic”} \]

\[ X(x) = \sin \left( \frac{\pi}{L} x \right) \quad \sin \left( \frac{2\pi}{L} x \right) \quad \sin \left( \frac{3\pi}{L} x \right) \quad \sin \left( \frac{4\pi}{L} x \right) \]

\[ T(t) = \sin (\omega_1 t + \phi_1) \quad \sin (\omega_2 t + \phi_2) \quad \sin (\omega_3 t + \phi_3) \quad \sin (\omega_4 t + \phi_4) \]

\[ \omega_1 = \frac{\pi v}{L} \quad \omega_2 = 2\omega_1 \quad \omega_2 = 3\omega_1 \quad \omega_2 = 4\omega_1 \]
Road map for today’s lecture

• Motivate a quantum mechanical wave equation by starting from a classical traveling wave and incorporating the de Broglie relation

• Separate this Schrödinger quantum wave equation into time-dependent and time-independent components

• Solve the time-dependent Schrödinger equation

• Lay out our plans to solve the time-independent Schrödinger equation for particles on various potentials
Practice problem: probability density

The total wavefunction of a quantum mechanical stationary state can be written as:

\[ \Psi(x, t) = \psi(x) e^{-iEt/\hbar} \]

We will learn that the probability density for where we expect to find a quantum particle along the \( x \) axis at time \( t \) is given by:

\[ |\Psi(x, t)|^2 = \Psi^*(x, t) \cdot \Psi(x, t) \]

What is the time dependence of this quantity for our stationary state? What does this imply?

Note: \( \Psi^*(x, t) \) represents the complex conjugate of \( \Psi(x, t) \)
Practice problem: probability density

\[
\int_{a}^{b} |\Psi(x, t)|^2 \, dx = \begin{cases} 
\text{probability of finding the particle between } a \text{ and } b, \text{ at time } t
\end{cases}
\]
Wrapping up

• **Next time:** the quantum free particle and particle-in-a-box

• **Today’s reading:** McQuarrie Chap. 3

• **Problem Set 1:** posted on Canvas website, due next Tuesday at 5 PM ET

• **In the last minute or two:** think about your muddiest point from this lecture. What was confusing, what lingering questions do you have?
We learned that if $\psi_n(x)$ is a stationary state (e.g. an eigenfunction of the time-independent Schrödinger equation) then its full time-dependent form is:

$$\Psi(x, t) = \psi_n(x)e^{-iE_n t/\hbar}$$

Show that if $\psi_m(x)$ and $\psi_n(x)$ are both stationary states, then the state

$$\Psi(x, t) = c_m \psi_m(x)e^{-iE_m t/\hbar} + c_n \psi_n(x)e^{-iE_n t/\hbar}$$

satisfies the time-dependent Schrödinger equation.