## Fabry-Pérot Cavity Cheatsheet

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### Cavity Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>( L )</td>
<td>cavity length</td>
</tr>
<tr>
<td>( r_1, r_2 )</td>
<td>reflection amplitude coefficients of mirrors 1 and 2</td>
</tr>
<tr>
<td>( R \equiv r_1^2 + r_2^2 )</td>
<td>reflection intensity coefficient per mirror, assuming mirrors 1 and 2 are identical</td>
</tr>
<tr>
<td>( t_1, t_2 )</td>
<td>transmission amplitude coefficients of mirrors 1 and 2</td>
</tr>
<tr>
<td>( T \equiv t_1^2 + t_2^2 )</td>
<td>transmission intensity coefficient per mirror, assuming mirrors 1 and 2 are identical</td>
</tr>
<tr>
<td>( \ell \equiv 1 - T - R )</td>
<td>loss intensity coefficient per mirror bounce</td>
</tr>
<tr>
<td>( \delta = 4\pi L\nu/c )</td>
<td>phase shift accumulated over one cavity round trip</td>
</tr>
<tr>
<td>( \alpha(\nu) )</td>
<td>frequency-dependent absorption coefficient of intracavity material</td>
</tr>
<tr>
<td>( n(\nu) )</td>
<td>frequency-dependent refractive index of intracavity material</td>
</tr>
</tbody>
</table>

### Cavity Light Expressions

#### Amplitude

| Reflection | \( \frac{E_R}{E_0} = \frac{r_2 (t_2^2 + r_1^2) e^{-\alpha L} e^{-i\delta}}{1 - r_1 r_2 e^{-\alpha L} e^{-i\delta}} \)
| Circulating | \( \frac{E_C}{E_0} = \frac{t_1}{1 - r_1 r_2 e^{-\alpha L} e^{-i\delta}} \)
| Transmitted | \( \frac{E_T}{E_0} = \frac{t_1 t_2 e^{-\alpha L/2}}{1 - r_1 r_2 e^{-\alpha L} e^{-i\delta}} \)
| Absorbed | \( \frac{I_A}{I_0} = 1 - \frac{I_R}{I_0} - \frac{I_T}{I_0} \)

#### Intensity

| Reflection | \( \frac{|E_R|^2}{I_0} = \frac{R + R(R+T)^2 e^{-2\alpha L - 2R(R+T)e^{-\alpha L} \cos(4\pi L\nu/c)}}{1 + R^2 e^{-2\alpha L - 2Re^{-\alpha L} \cos(4\pi L\nu/c)}} \)
| Circulating | \( \frac{|E_C|^2}{I_0} = \frac{T}{1 + R^2 e^{-2\alpha L - 2Re^{-\alpha L} \cos(4\pi L\nu/c)}} \)
| Transmitted | \( \frac{|E_T|^2}{I_0} = \frac{T^2 e^{-\alpha L}}{1 + R^2 e^{-2\alpha L - 2Re^{-\alpha L} \cos(4\pi L\nu/c)}} \)
| Absorbed | \( \frac{I_A}{I_0} = 1 - \frac{I_R}{I_0} - \frac{I_T}{I_0} \)

### Empty-Cavity Figures of Merit

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_m = \frac{c}{2nL} \cdot m )</td>
<td>( m )th cavity fringe resonance position (Hz)</td>
</tr>
<tr>
<td>FSR = ( \frac{c}{2nL} )</td>
<td>free spectral range (Hz)</td>
</tr>
<tr>
<td>( \mathcal{F} = \frac{\pi \sqrt{TT_2}}{1 - r_1 r_2} = \frac{\pi \sqrt{F}}{1 - R} )</td>
<td>finesse</td>
</tr>
<tr>
<td>( \Delta \nu = \frac{\text{FSR}}{\mathcal{F}} )</td>
<td>linewidth, fwhm (Hz)</td>
</tr>
<tr>
<td>( \tau_c = \frac{[2\pi \Delta \nu]^{-1}}{c} )</td>
<td>cavity photon ringdown lifetime (s)</td>
</tr>
<tr>
<td>( Q = \frac{\nu}{\Delta \nu} = \frac{\nu \text{FSR}}{F} = \frac{2nL F \nu}{c} = \frac{2nL}{\lambda} \mathcal{F} )</td>
<td>Q-factor (finesse times wavelengths per round trip)</td>
</tr>
<tr>
<td>( \frac{I_T(\alpha=0)}{I_0} = \frac{T^2}{(1-R)^2} = \frac{T^2}{(T+\ell)^2} )</td>
<td>cavity transmission fraction on resonance</td>
</tr>
<tr>
<td>( \frac{\beta \mathcal{F}}{\pi} )</td>
<td>effective cavity pathlength enhancement</td>
</tr>
</tbody>
</table>

\( \beta = 2 \), cw cavity-coupling; \( \beta = 1 \), pulsed light