Fabry-Pérot Cavity Cheatsheet

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Cavity Parameters

L	cavity length
r_1, r_2	reflection *amplitude* coefficients of mirrors 1 and 2
$R \equiv r_1^2 = r_2^2$	reflection *intensity* coefficient per mirror, assuming mirrors 1 and 2 are identical
t_1, t_2	transmission *amplitude* coefficients of mirrors 1 and 2
$T \equiv t_1^2 = t_2^2$	transmission *intensity* coefficient per mirror, assuming mirrors 1 and 2 are identical
$\ell \equiv 1 - T - R$	loss intensity coefficient per mirror bounce
$\delta = 4\pi L n \nu / c$	phase shift accumulated over one cavity round trip
$\alpha(\nu)$	frequency-dependent absorption coefficient of intracavity material
$n(\nu)$	frequency-dependent refractive index of intracavity material

Cavity Light Expressions

	Amplitude	Intensity
Reflected	$\frac{E_R}{E_0} = \frac{r_2\,(t_1^2+r_1^2)\,e^{-\alpha L}\,e^{-i\delta} - r_1}{1 - r_1\,r_2\,e^{-\alpha L}\,e^{-i\delta}}$	$\frac{I_R}{I_0} = \left\lvert \frac{E_R}{E_0} \right\rvert^2 = \frac{R + R(R+T)^2 e^{-2\alpha L} - 2R(R+T)e^{-\alpha L}\cos(4\pi L n\nu/c)}{1 + R^2 e^{-2\alpha L} - 2Re^{-\alpha L}\cos(4\pi L n \nu/c)}$
Circulating	$\frac{E_C}{E_0} = \frac{t_1}{1 - r_1\,r_2\,e^{-\alpha L}\,e^{-i\delta}}$	$\frac{I_C}{I_0} = \left\lvert \frac{E_C}{E_0} \right\rvert^2 = \frac{T}{1 + R^2 e^{-2\alpha L} - 2Re^{-\alpha L}\cos(4\pi L n \nu/c)}$
Transmitted	$\frac{E_T}{E_0} = \frac{t_1\,t_2\,e^{-\alpha L/2}}{1 - r_1\,r_2\,e^{-\alpha L}\,e^{-i\delta}}$	$\frac{I_T}{I_0} = \left\lvert \frac{E_T}{E_0} \right\rvert^2 = \frac{T^2 e^{-\alpha L}}{1 + R^2 e^{-2\alpha L} - 2Re^{-\alpha L}\cos(4\pi L n \nu/c)}$
Absorbed		$\frac{I_A}{I_0} = 1 - \frac{I_R}{I_0} - \frac{I_T}{I_0}$

Empty-Cavity Figures of Merit

$\nu_m = \frac{c}{2nL} \cdot m$	m^{th} cavity fringe resonance position (Hz)
$\mathrm{FSR} = \frac{c}{2nL}$	free spectral range (Hz)
$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2} = \frac{\pi\sqrt{R}}{1-R}$	finesse
$\Delta\nu = \frac{\mathrm{FSR}}{\mathcal{F}}$	linewidth, fwhm (Hz)
$\tau_c = [2\pi\,\Delta\nu]^{-1}$	cavity photon ringdown lifetime (s)
$\mathcal{Q} = \frac{\nu}{\Delta\nu} = \frac{\nu\mathcal{F}}{\mathrm{FSR}} = \frac{2nL\mathcal{F}\nu}{c} = \frac{2nL}{\lambda}\mathcal{F}$	Q-factor (finesse times wavelengths per round trip)
$\frac{I_T(\alpha=0)}{I_0} = \frac{T^2}{(1-R)^2} = \frac{T^2}{(T+\ell)^2}$	cavity transmission fraction on resonance
$\frac{\beta\mathcal{F}}{\pi}$	effective cavity pathlength enhancement $\beta = 2$, cw cavity-coupling; $\beta = 1$, pulsed light