A particle in a simple harmonic oscillator (HO) potential energy well $V(x) = \frac{1}{2}kx^2$ has an initial wavefunction in the second excited state of the HO: $\Psi(x, t = 0) = \Psi_2(x)$.

Choose all of the statements that are correct.

A. The expectation value of the <u>position</u> of this particle depends on time *t*

B. The expectation value of the <u>momentum</u> of this particle depends on time *t*

C. The expectation value of the <u>energy</u> of this particle depends on time *t*

D. None of the above

# CHM 305 The Quantum World

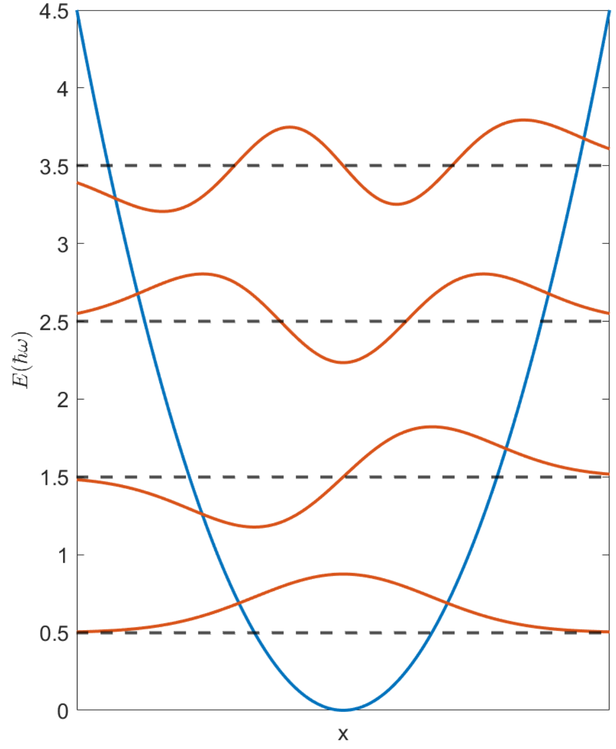
## Lecture 10: Angular Momentum

McQuarrie Ch. 6

# Last time we discussed the quantum harmonic oscillator

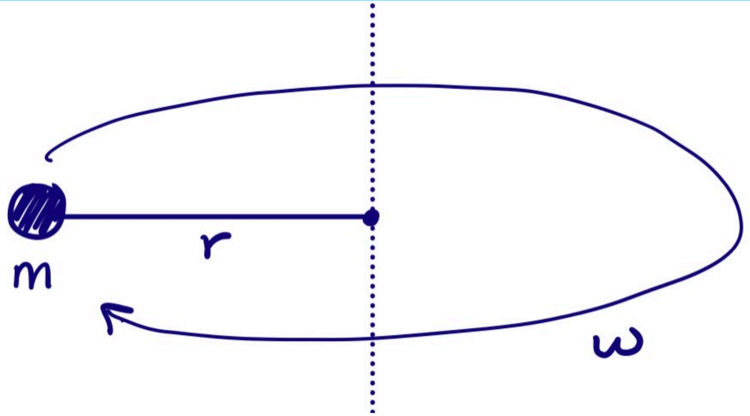
- Briefly lay out what we know about the classical harmonic oscillator
- Introduce the quantum harmonic oscillator, and its relevance to molecular vibrations
- Write down the Schrödinger equation for this system, and examine the resulting wavefunctions and energy eigenvalues
- Learn about some nice properties of these wavefunctions


$$V(x) = \frac{1}{2}kx^2$$



# Road map for today: angular momentum

- Lay out definitions for classical circular motion and angular momentum



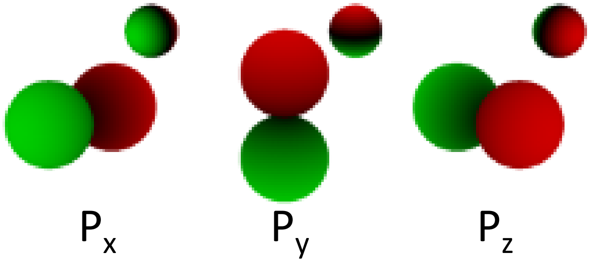

- Discuss rotations of classical and quantum mechanical rigid bodies

- Write down the Schrodinger equation and its solutions for the quantum rigid rotor in spherical coordinates

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2$$

$$\hat{H}\psi(\theta,\phi) = E\psi(\theta,\phi)$$

- Make connections to rotations of diatomic molecules and the hydrogen atom



# Practice Problem #1:

The rigid rotor Hamiltonian has the form

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\right] + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

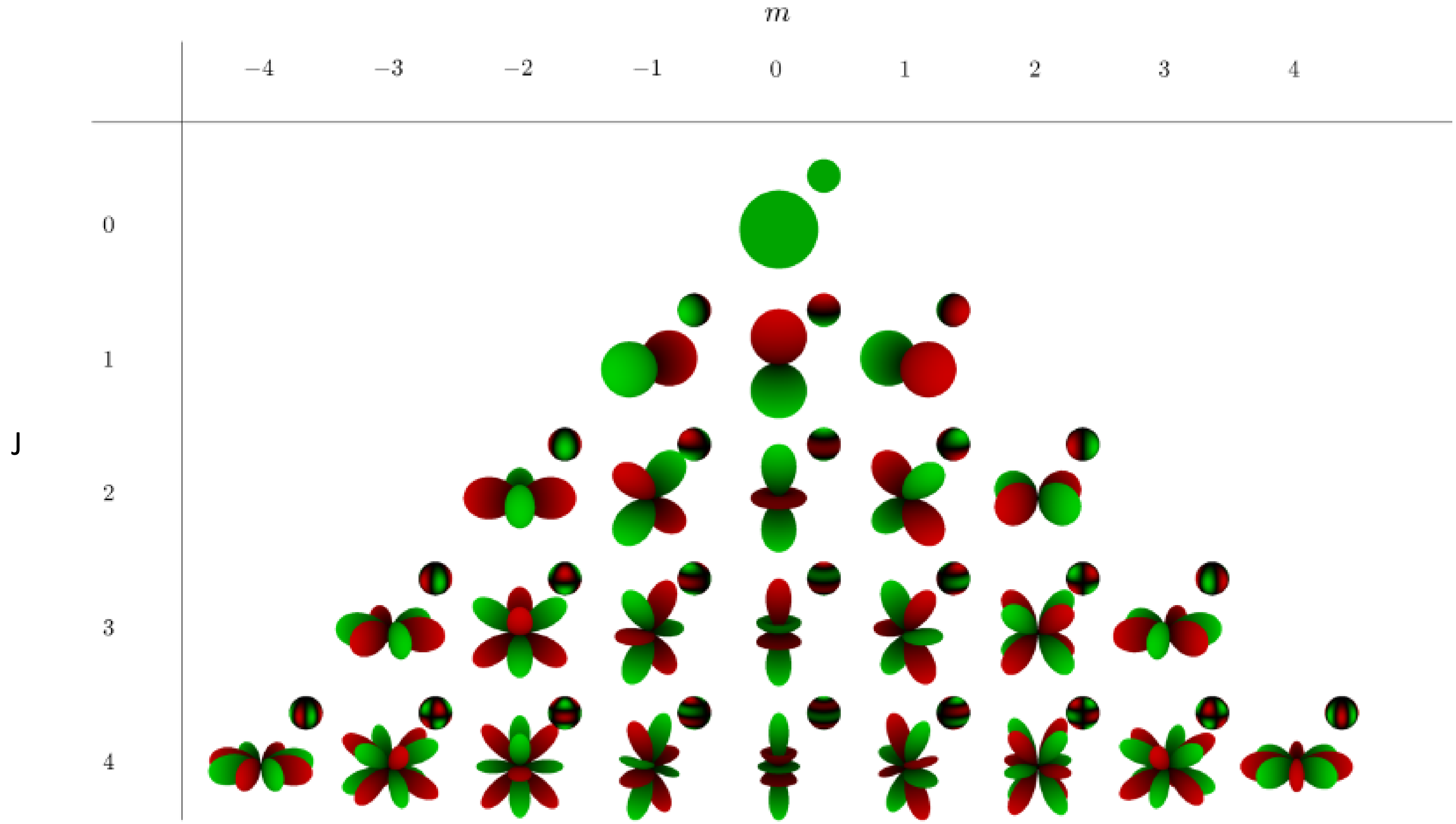
and can also be written as

$$\hat{H} = \frac{\hat{\ell}^2}{2I}$$

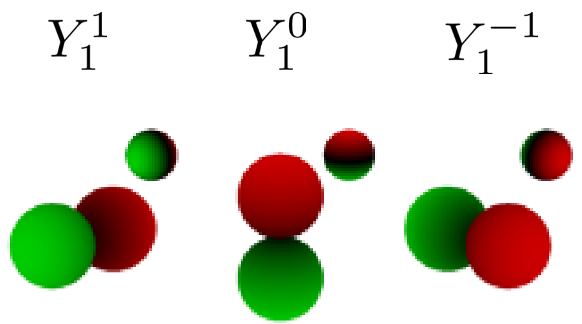
Do $\hat{H}$ and $\hat{\ell}^2$ commute? What does this tell you about their eigenfunctions?

Can you draw an analogy to the energy and linear momentum of a free particle?

# Spherical Harmonics



# Practice Problem #2:




Show that

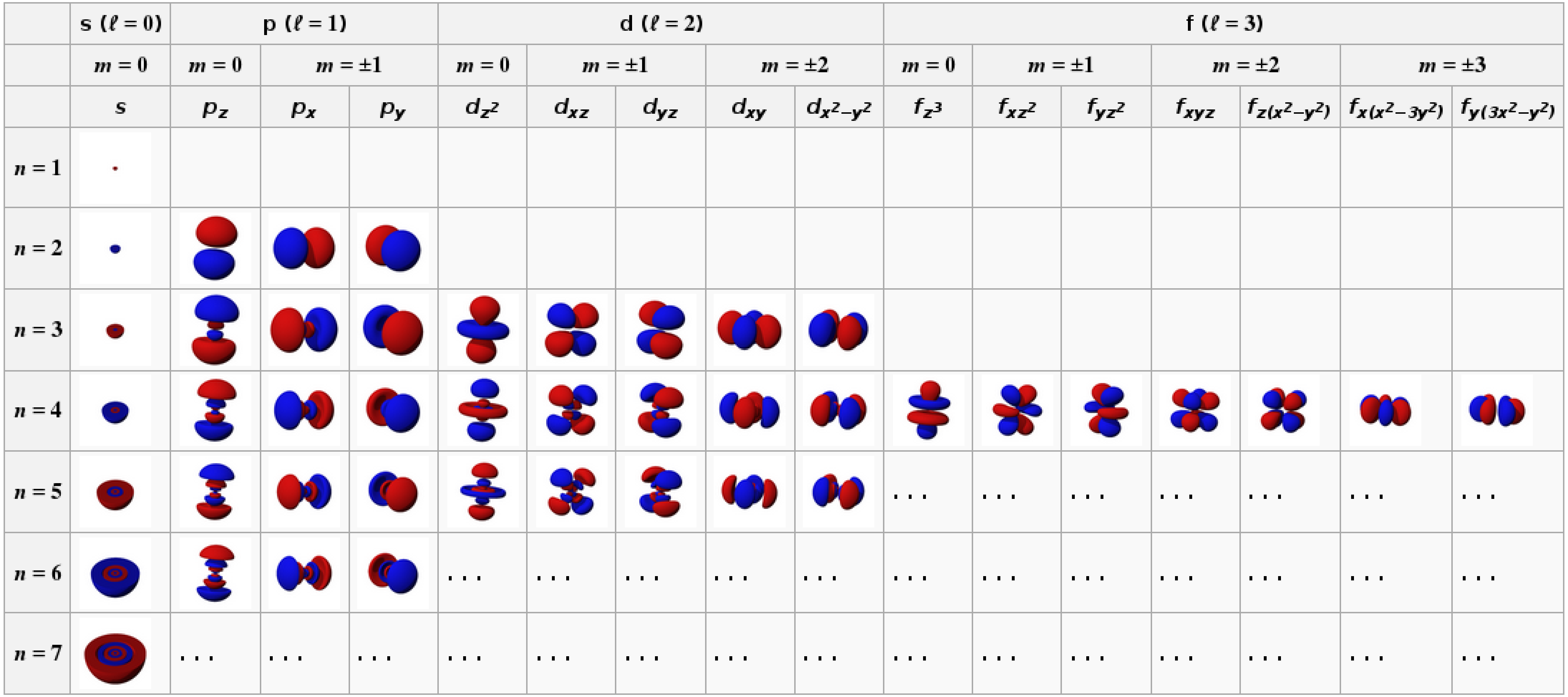
$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$$

is a solution to the rigid rotor Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\right] + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

and find its energy eigenvalue.

# Hydrogen atom orbitals




https://en.wikipedia.org/wiki/Atomic_orbital