CHM 305 The Quantum World
Lecture 3: The Schrödinger equation

Reading: McQuarrie Chap. 3

Quick review of what we learned last time…

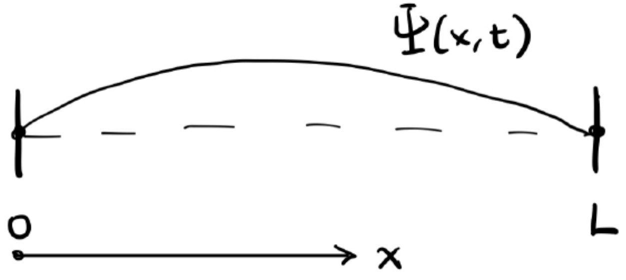
- Show how classical waves have *wavefunctions* and are solutions to the *classical wave equation*

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 \psi(x,t)}{\partial t^2}$$

- Solve the wave equation with separation of variables

- Show how the solutions to the wave equation with *boundary conditions* give us a set of standing wave solutions

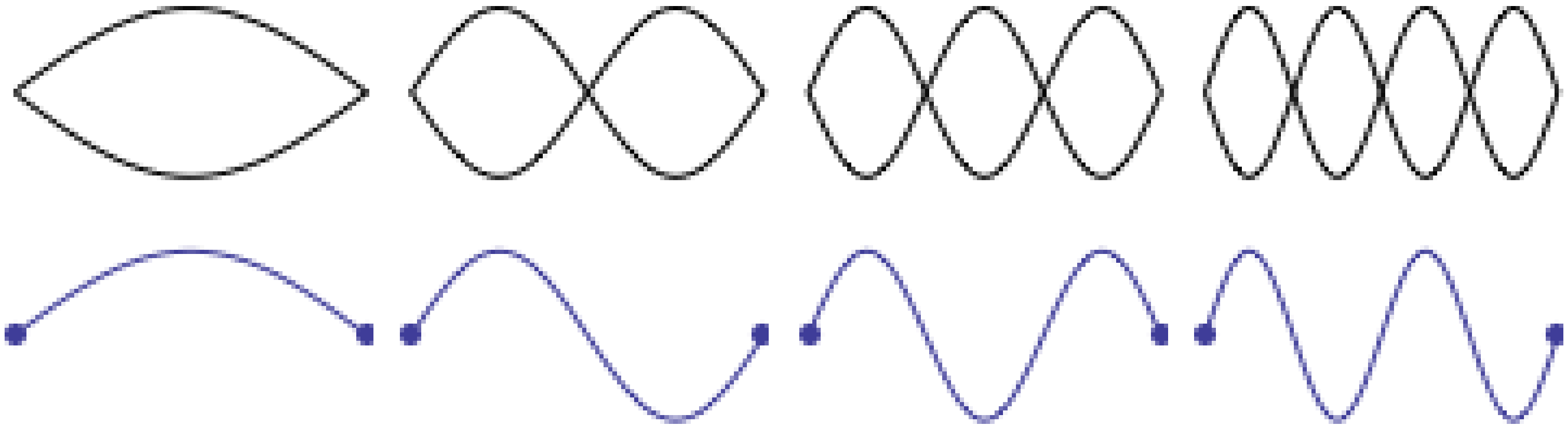
$$\psi(x,t) = X(x) \cdot T(t)$$



- Classical standing waves will be a nice analogy for solutions to Schrödinger's quantum wave equation

Standing wave normal modes

	$n=1$	$n=2$	$n=3$	$n=4$
	"first harmonic"	"second harmonic"	"third harmonic"	"fourth harmonic"



$$\Psi(x,t) = X(x)\cdot T(t)$$

	$n=1$	$n=2$	$n=3$	$n=4$
$X(x) =$	$\sin\left(\frac{\pi}{L}x\right)$	$\sin\left(\frac{2\pi}{L}x\right)$	$\sin\left(\frac{3\pi}{L}x\right)$	$\sin\left(\frac{4\pi}{L}x\right)$
$T(t) =$	$\sin\left(\omega_1 t + \phi_1\right)$	$\sin\left(\omega_2 t + \phi_2\right)$	$\sin\left(\omega_3 t + \phi_3\right)$	$\sin\left(\omega_4 t + \phi_4\right)$
	$\omega_1 = \frac{\pi v}{L}$	$\omega_2 = 2\omega_1$	$\omega_2 = 3\omega_1$	$\omega_2 = 4\omega_1$

Road map for today's lecture

- Motivate a *quantum mechanical wave equation* by starting from a classical traveling wave and plugging in the de Broglie wavelength

- Separate this Schrödinger quantum wave equation into time-dependent and time-independent components

- Solve the time-dependent Schrödinger equation

- Lay out our plans to solve the time-independent Schrödinger equation for particles in various environments

Practice problem 1: Probability density

The total wavefunction of a quantum mechanical stationary state can be written as:

$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$$

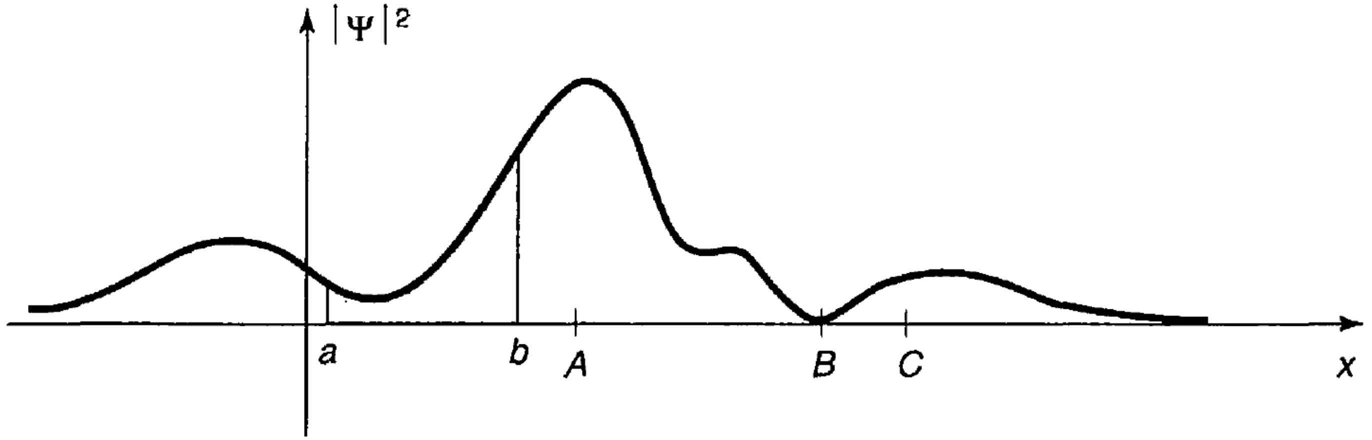
We will learn that the probability density for where we expect to find a quantum particle along the *x* axis at time *t* is given by:

$$|\Psi(x,t)|^2 = \Psi^*(x,t) \cdot \Psi(x,t)$$

How does the probability density of a stationary state evolve in time?

Note: $\Psi^*(x,t)$ represents the <u>*complex conjugate*</u> of $\Psi(x,t)$

Practice problem 1: Probability density



$$\int_a^b |\Psi(x,t)|^2 dx \quad = \quad \left\{ \text{probability of finding the particle between } a \text{ and } b \text{, at time } t \right.$$

Practice problem 2 : Superpositions

We learned that if $\psi_n(x)$ is a stationary state (e.g. a solution to the time-independent Schrödinger equation), then its full time-dependent form is:

$$\Psi(x,t) = \psi(x)\, e^{-iE_n t/\hbar}$$

1. Show that if $\psi_1(x)$ and $\psi_2(x)$ are both stationary states, then the state:

   $$\Phi(x,t) = c_1\psi_1(x)\, e^{-iE_1 t/\hbar} + c_2\psi_2(x)\, e^{-iE_2 t/\hbar}$$

   satisfies the full time-*dependent* Schrödinger equation ($\hat{H}\Phi = i\hbar\frac{\partial}{\partial t}\Phi$).

2. Does $\Phi(x,t)$ satisfy the time-*independent* Schrödinger equation ($\hat{H}\Phi = E\Phi$)? Why or why not?