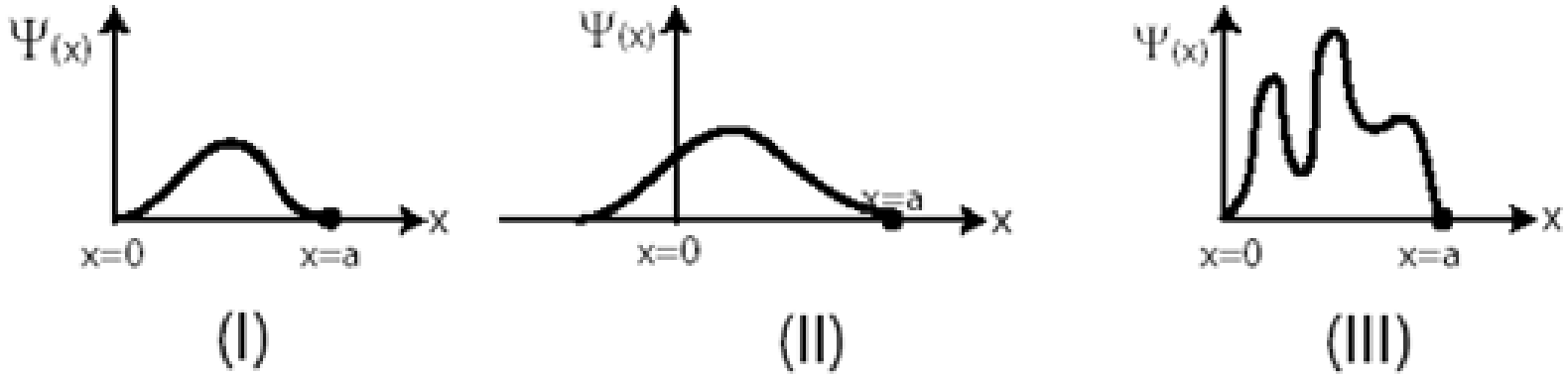
Choose all of the following wavefunctions that are valid wavefunctions for a 1D particle-in-a-box of width a (boundaries between $x = 0$ and $x = a$) at time $t = 0$:



A. (I) only
B. (I) and (II)
C. (I) and (III)
D. (II) and (III)
E. All of the above

CHM 305 The Quantum World

Lecture 6: The Rules of Quantum Mechanics (pt. 2)

Reading: McQuarrie Ch. 4

Last lecture we discussed…

Quantum operators:

- are linear

$$\hat{A}\sum_{i=1}^{\infty} c_i\psi_i = \sum_{i=1}^{\infty} c_i a_i \psi_i$$

- May or may not commute

$$\hat{A}\hat{B}\psi(x) \neq \hat{B}\hat{A}\psi(x)$$

Eigenfunctions of an operator:

- are normalized

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2 dx = 1$$

- are mutually orthogonal

$$\int_{-\infty}^{\infty} \psi_n^*(x)\psi_m(x)dx = 0$$

- form a complete set

$$\Phi(x) = \sum_{n=1}^{\infty} b_n \psi_n(x)$$

Road map for today's lecture

Discuss in detail the five *Postulates of Quantum Mechanics*

1. Wave functions represent probability distributions
2. Operators represent physical quantities, or observables
3. Measurements with operators read out eigenvalues
4. How to calculate expectation values of measurements
5. The time-dependent Schrödinger equation

Practice problem #1

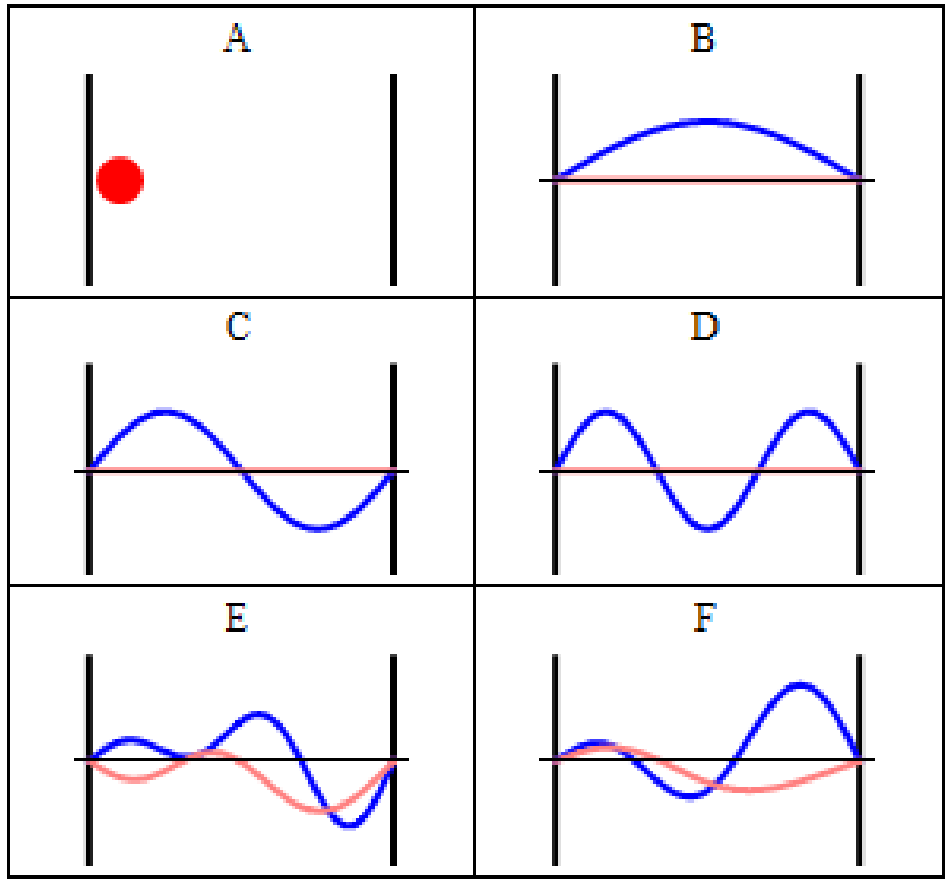
Consider the superposition state

$$\phi(x) = \frac{\sqrt{3}}{2}\psi_1(x) + \frac{i}{2}\psi_2(x)$$

where $\psi_n(x)$ is the eigenfunction of the $\hat{A}$ operator with eigenvalue a_n.

- What are the possible values we would measure when applying $\hat{A}$ to $\phi(x)$?
- What are the probabilities of making each of those measurements?

Particle in a box time-dependence



https://en.wikipedia.org/wiki/Particle_in_a_box

Blue ~ real part of wavefunction

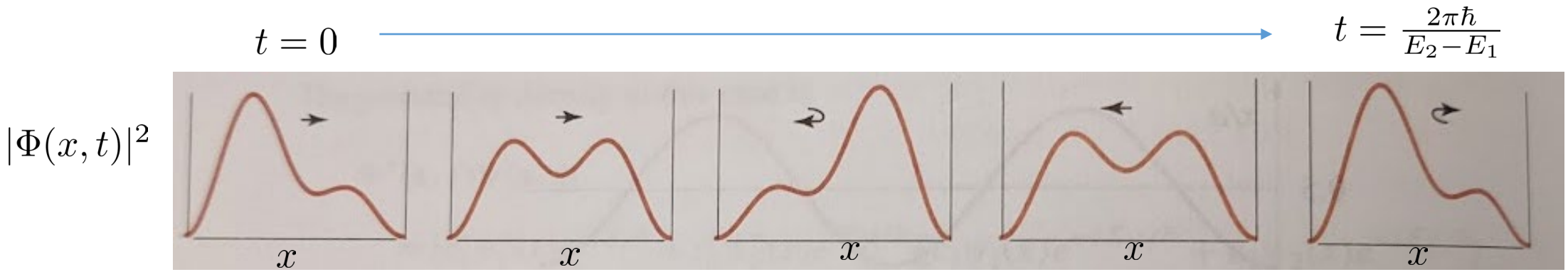
Pink ~ imaginary part of wavefunction

- B, C, D are stationary states (standing waves)
- E, F are random superposition states of the four lowest energy eigenstates

Particle in a box time-dependence

Another example (from McQuarrie, Ch. 4, Figure 4.4)

$$\Phi(x,t) = \left(\frac{1}{a}\right)^{1/2} e^{-iE_1 t/\hbar} \sin\left(\frac{\pi x}{a}\right) + \left(\frac{1}{a}\right)^{1/2} e^{-iE_2 t/\hbar} \sin\left(\frac{2\pi x}{a}\right)$$



Practice problem #2

What is the expected value of the energy of the particle-in-a-box superposition state below?

$$\Phi(x,t) = \frac{1}{\sqrt{2}}\Psi_1(x,t) + \frac{1}{\sqrt{2}}\Psi_2(x,t)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{2}{a}\right)^{1/2}\sin\left(\frac{\pi x}{a}\right)e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\left(\frac{2}{a}\right)^{1/2}\sin\left(\frac{2\pi x}{a}\right)e^{-iE_2 t/\hbar}$$