Consider the temporal dynamics of $\Psi(x, t)$ when we initialize the system at time $t = 0$ in the superposition state

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}} [\psi_1(x) + \psi_2(x)]$$

where $\psi_1(x)$ and $\psi_2(x)$ are two eigenstates of the 1D particle in a box. Which of the following statements are correct? Select all that apply.

(a) The probability distribution $|\Psi(x, t)|^2$ has time-dependent behavior

(b) The expectation value for the position of the particle

$$\langle x \rangle = \int_{-\infty}^{\infty} dx \; \Psi^* (x, t) \cdot \hat{x} \cdot \Psi (x, t)$$

has time-dependent behavior

(c) The expectation value for the energy of the particle

$$\langle E \rangle = \int_{-\infty}^{\infty} dx \; \Psi^* (x, t) \cdot \hat{H} \cdot \Psi (x, t)$$

has time-dependent behavior
The five *Postulates of Quantum Mechanics*

1. Wave functions represent probability distributions
   \[ P = \int_{x_1}^{x_2} |\Psi(x, t)|^2 dx \]

2. Operators represent physical quantities, or observables
   \[ x \rightarrow \hat{x} \]
   \[ E \rightarrow \hat{H} \]

3. Measurements with operators read out eigenvalues
   \[ \hat{H} \sum_n c_n \psi_n(x) \rightarrow E_n \text{ with probability } |c_n|^2 \]

4. Expectation values of measurements
   \[ \langle E \rangle = \int_{-\infty}^{\infty} \phi^*(x) \hat{H} \phi(x) dx \]

5. The time-dependent SE
   \[ \hat{H} \Psi(x, t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t} \]
We will find wavefunction solutions for several problems involving potentials with finite walls

- Particle in a finite box
- Particle hitting a finite step
- Particle tunneling through a finite barrier
- How quantum tunneling can be important to chemical reactions
Practice Problem #1: Finite step potential

Qualitatively, what do you expect to happen for an incoming particle with $E < V_0$?

Use what we learned about the particle in a finite box to write down general expressions for the three pieces of the total wavefunction:

1. Incoming wave
2. Reflected wave
3. Transmitted wave

What are the relevant boundary conditions you would use to constrain these general expressions?
Practice Problem #2: A particle and a cliff

- Qualitatively, what do you expect to happen for a particle incoming from the left that hits a cliff where $V(x)$ drops from 0 to $-V_0$?

- What would the wavefunctions look like in regions 1 and 2?

- What are the relevant boundary conditions?