Consider the temporal dynamics of $\Psi(x,t)$ when we initialize the system at time $t = 0$ in the superposition state

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}} [\psi_1(x) + \psi_2(x)]$$

where $\psi_1(x)$ and $\psi_2(x)$ are two eigenstates of the 1D particle in a box. Which of the following statements are correct? **Select all that apply.**

(a) The probability distribution $|\Psi(x,t)|^2$ has time-dependent behavior

(b) The expectation value for the position of the particle $\langle x \rangle = \int_{-\infty}^{\infty} dx\ \Psi^*(x,t) \cdot \hat{x} \cdot \Psi(x,t)$ has time-dependent behavior

(c) The expectation value for the energy of the particle $\langle E \rangle = \int_{-\infty}^{\infty} dx\ \Psi^*(x,t) \cdot \hat{H} \cdot \Psi(x,t)$ has time-dependent behavior

CHM 305 The Quantum World

Lecture 7: Quantum Tunneling

Last lecture we discussed…

The five *Postulates of Quantum Mechanics*

1. Wave functions represent probability distributions $P = \int_{x_1}^{x_2} |\Psi(x,t)|^2 dx$

2. Operators represent physical quantities, or observables $x \to \hat{x}$ $E \to \hat{H}$

3. Measurements with operators read out eigenvalues $\hat{H} \sum_n c_n \psi_n(x) \to$ E_n with probability $|c_n|^2$

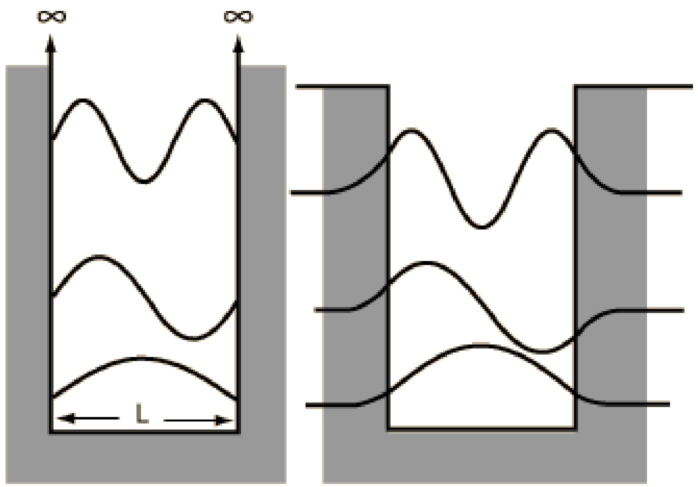
4. Expectation values of measurements $\langle E \rangle = \int_{-\infty}^{\infty} \phi^*(x) \hat{H} \phi(x) dx$

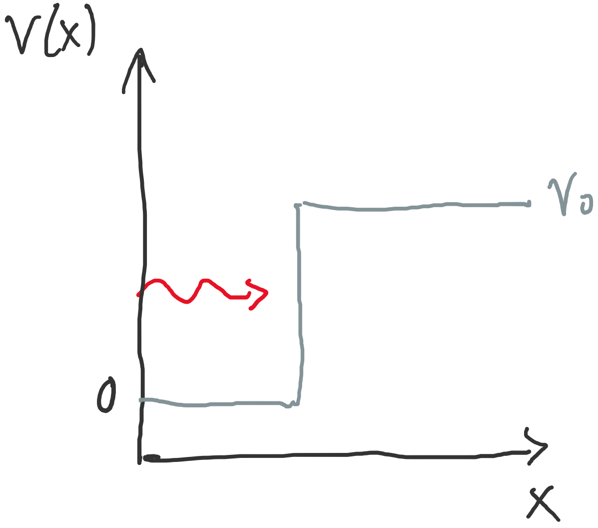
5. The time-dependent SE $\hat{H}\Psi(x,t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

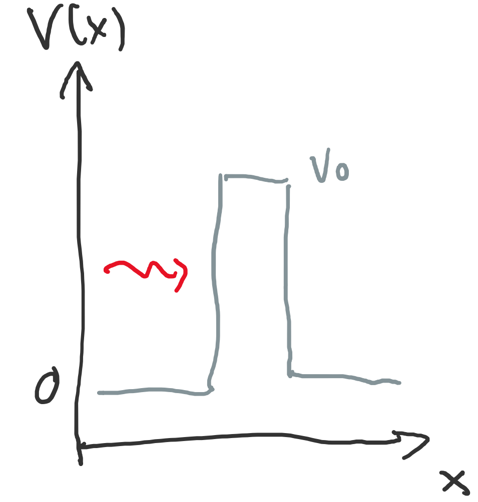
Road map for today's lecture

We will find wavefunction solutions for several problems involving potentials with *finite* walls

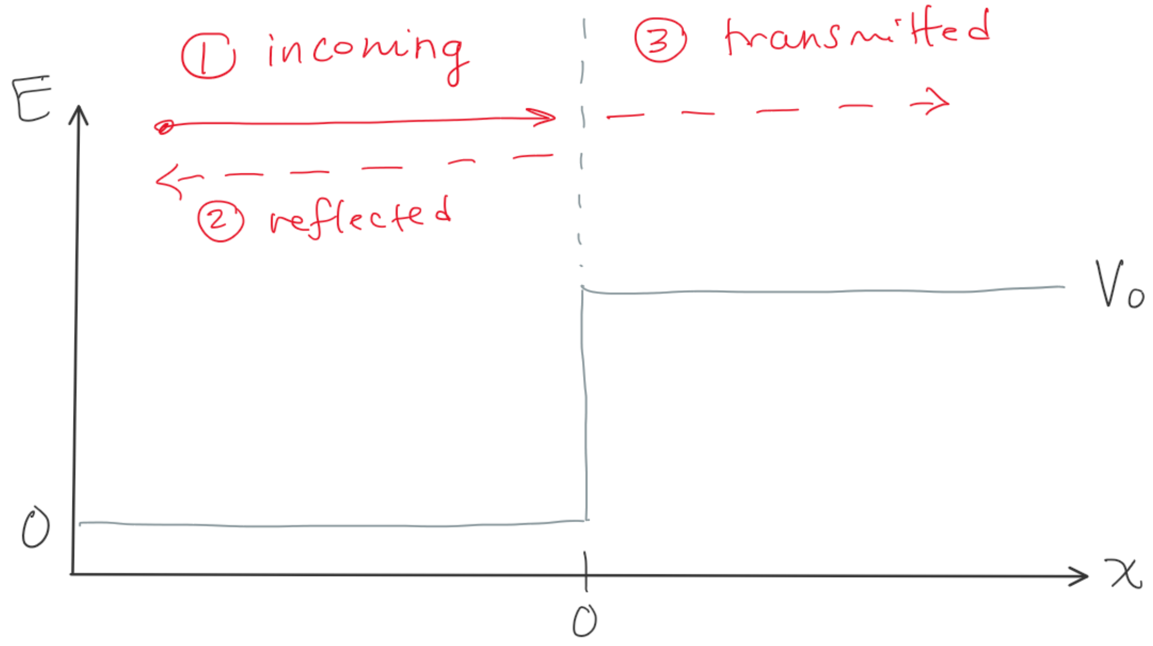
- Particle in a finite box
- Particle hitting a finite step
- Particle tunneling through a finite barrier
- How quantum tunneling can be important to chemical reactions







Practice Problem #1: Finite step potential



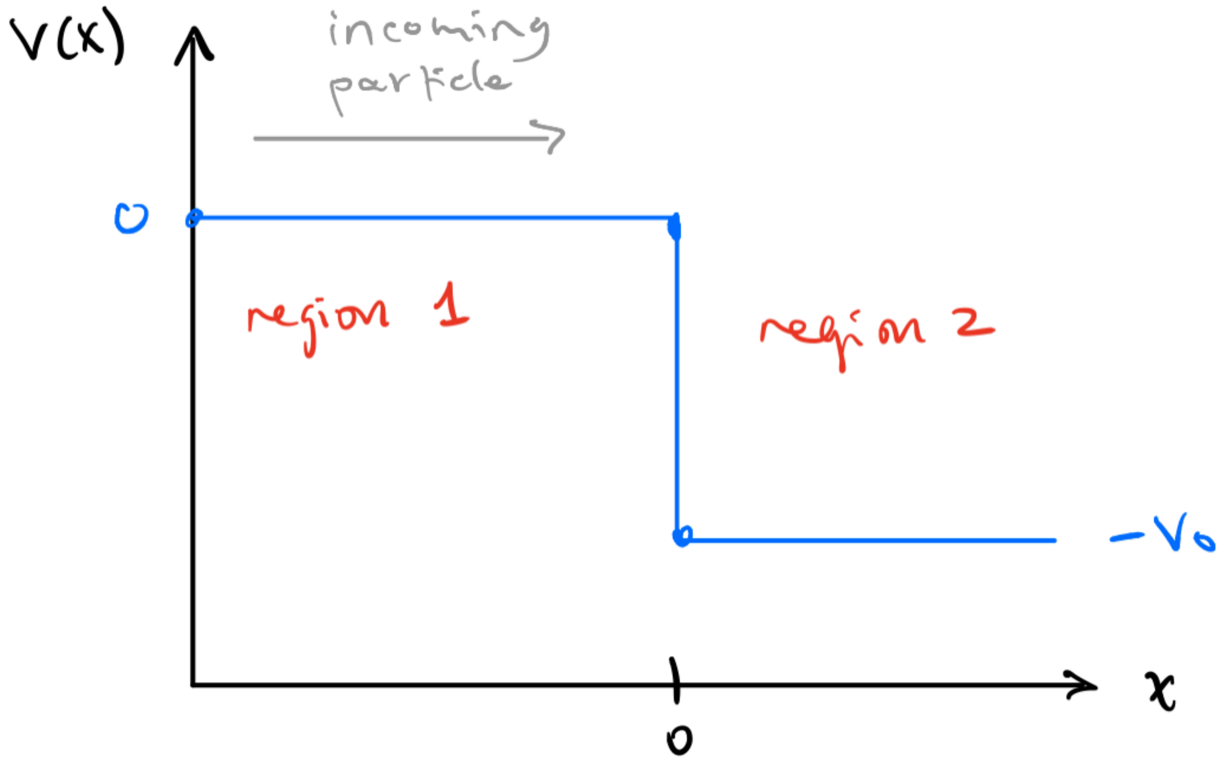
Qualitatively, what do you expect to happen for an incoming particle with **$E < V_0$**?

Use what we learned about the particle in a finite box to write down general expressions for the three pieces of the total wavefunction:

1. Incoming wave
2. Reflected wave
3. Transmitted wave

What are the relevant boundary conditions you would use to constrain these general expressions?

Practice Problem #2: A particle and a cliff



- Qualitatively, what do you expect to happen for a particle incoming from the left that hits a cliff where V(x) drops from 0 to $-V_0$?
- What would the wavefunctions look like in regions 1 and 2?
- What are the relevant boundary conditions?