The initial state of a 1D PIB is

$$\Psi(x,0) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + \psi_2(x)\right]$$

where $\psi_1(x)$ and $\psi_2(x)$ are the ground and first-excited PIB energy eigenstates. You measure the energy of the particle at time $t = 0$ and obtain E_1. Then immediately following the energy measurement at time $t = \delta t$, you measure the position of the particle.

What is the probability that you find the particle in the region between x_0 and $x_0 + dx$?

(a) $|\Psi(x_0, \delta t)|^2 dx$

(b) $x|\Psi(x_0, \delta t)|^2 dx$

(c) $|\psi_1(x_0)|^2 dx$

(d) $x|\psi_1(x_0)|^2 dx$

(e) None of the above

CHM 305 The Quantum World

Lecture 9: The Harmonic Oscillator

McQuarrie Ch. 5

Last time we discussed the uncertainty principle

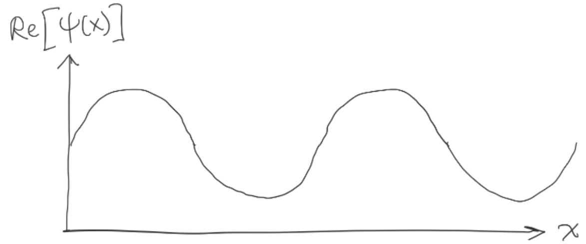
- The rules for quantum measurements and about “wavefunction collapse”

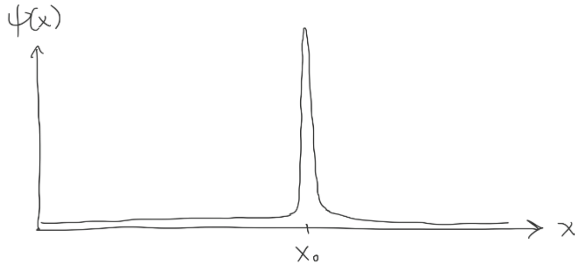
- Trade-offs in uncertainty between knowing a free particle’s momentum and position

- The variance and standard deviation as metrics for the uncertainty of a measurement

- Heisenberg’s uncertainty principle

- Consequences for operators that do and do not commute





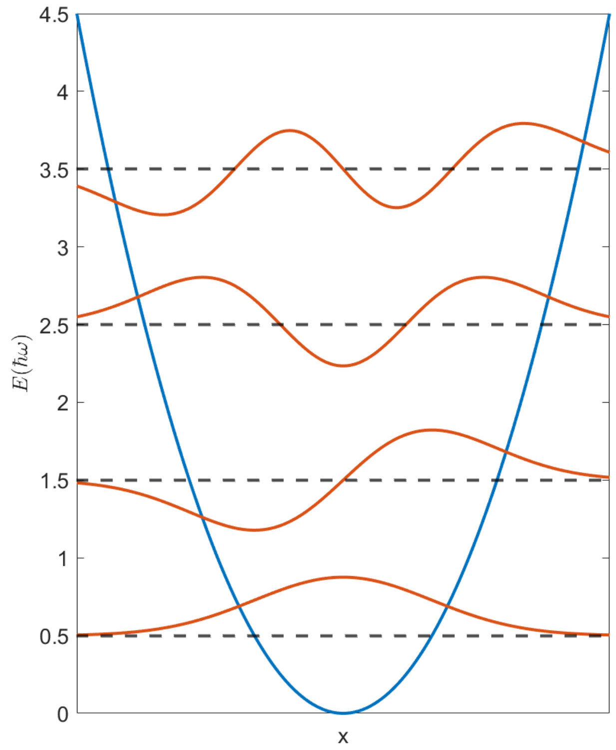
$$\sigma_A^2 = \langle A^2 \rangle - \langle A \rangle^2$$

$$\sigma_A^2 \sigma_B^2 \geq -\frac{1}{4}\left(\int \psi^* \, [\hat{A}, \hat{B}] \, \psi \, dx \right)^2$$

Road map for today's lecture…

- Briefly lay out what we know about the classical harmonic oscillator
- Introduce the quantum harmonic oscillator, and its relevance to molecular vibrations
- Write down the Schrödinger equation for this system, and examine the resulting wavefunctions and energy eigenvalues
- Learn about some nice properties of these wavefunctions

$V(x) = \frac{1}{2}kx^2$



Practice Problem #1: quantum HO solutions

Consider the quantum HO wavefunction with quantum number $n = 0$:

$$\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

Where $\alpha = \sqrt{\frac{k\mu}{\hbar^2}}$.

Show that this wavefunction solves the Schrödinger equation

$$\hat{H}\psi_n(x) = -\frac{\hbar^2}{2\mu}\frac{d^2\psi_n(x)}{x^2} + \frac{kx^2}{2}\psi_n(x) = E_n\psi_n(x)$$

and find the corresponding energy eigenvalue E_0.

Practice Problem #2: Even and odd functions

Determine whether the following functions are even, odd or neither:

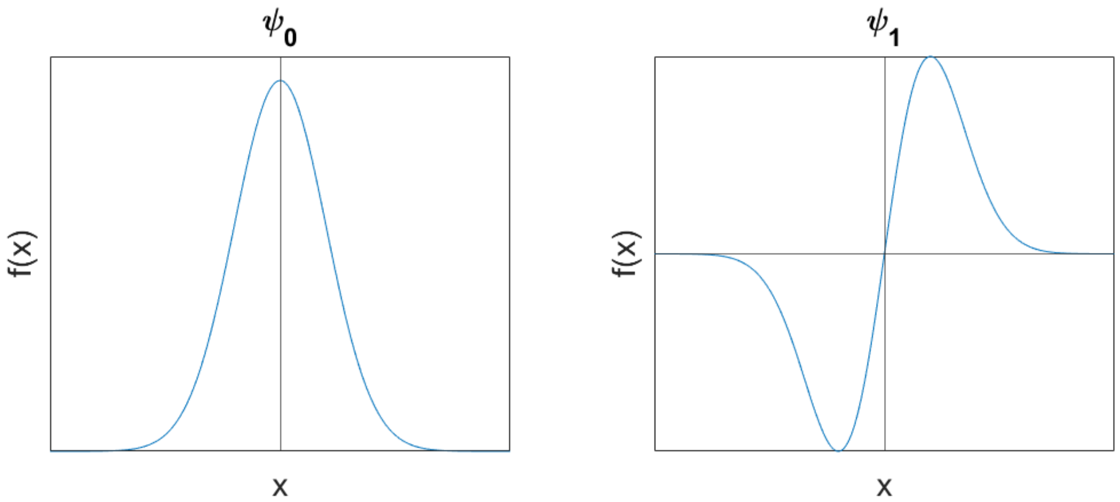
(a) $x \sin(x)$

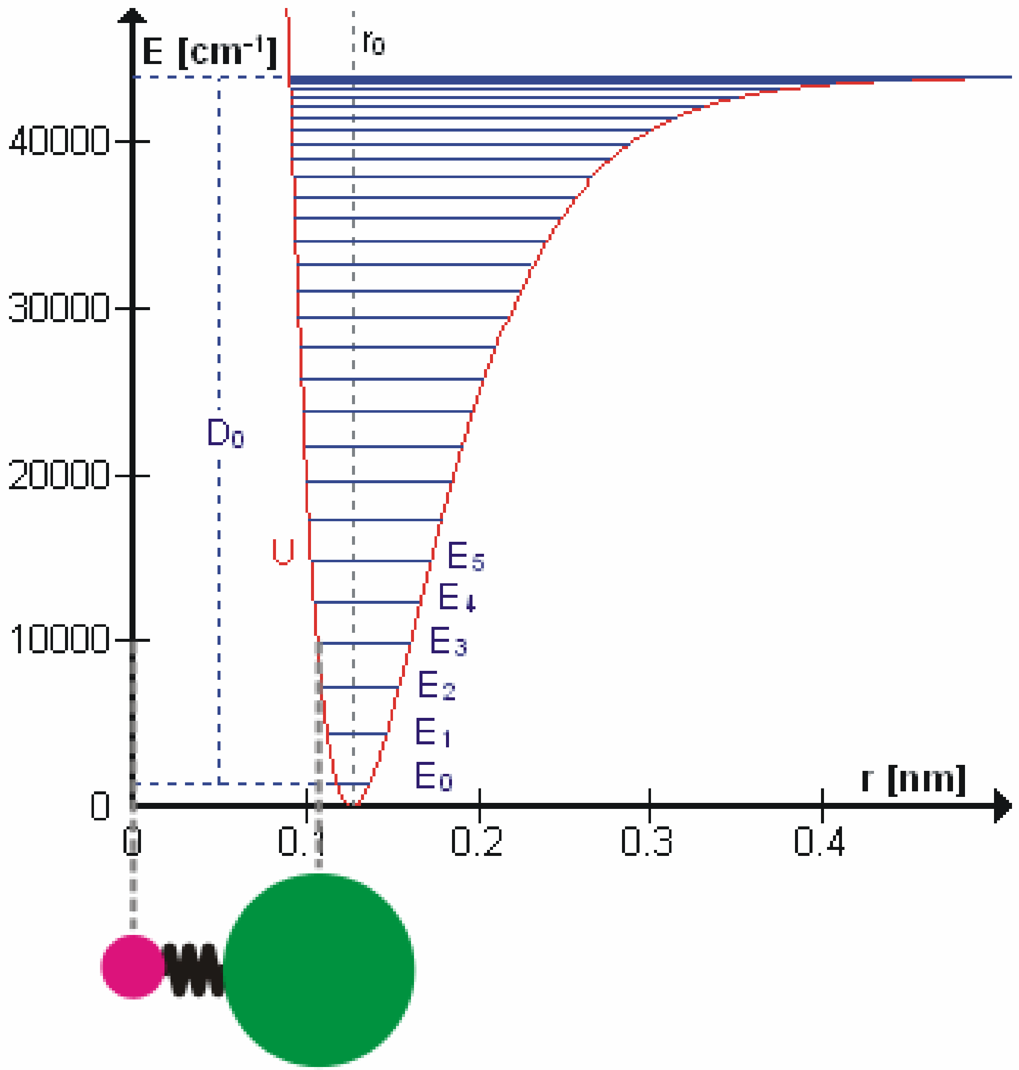
(b) $e^{-x^2} \cos(x)$

(c) $x^2 \sin(x)$

(d) e^{ix}

(e) What about the first two harmonic oscillator wavefunctions?





https://en.wikipedia.org/wiki/Molecular_vibration