# Problem Set 1
# CHM 305, Fall 2023

**Distributed:** Thursday, September 7, 2023
**Due:** Thursday, September 14, 2023 @ 5 PM ET

**Problem Set Submission:** Please submit assignment by uploading your files to Canvas. You may upload scanned handwritten work or digital documents as .pdf files. Please see the Submitting Assignments in Canvas page for detailed instructions on how best to do this.

**Collaboration:** Students are encouraged to interact with one another and to collaborate in learning and understanding the course material and homework problems. However, each student's assignments are expected to be their own work, reflecting their own understanding of the course material.

## 1 Properties of Light

(a) Radiation in the ultraviolet (UV) region of the electromagnetic spectrum is usually described in terms of wavelength, $\lambda$, and is given in nanometers ($10^{-9}$ m). Calculate the values of frequency ($\nu$) in Hz, wavenumber ($\tilde{\nu} = 1/\lambda$) in $\text{cm}^{-1}$, and energy ($E = h\nu$) in Joules for UV light with $\lambda = 200\,\text{nm}$.

(b) Radiation in the infrared (IR) region is often expressed in terms of wavenumbers. Calculate the values of $\nu$, $\lambda$, and $E$ for IR light with $\tilde{\nu} = 10^3\ \text{cm}^{-1}$.

## 2 de Broglie Wavelength

Calculate the de Broglie wavelength, $\lambda_d$ for:

(a) An electron with a kinetic energy of $100\,\text{eV}$.

(b) A proton with a kinetic energy of $100\,\text{eV}$.

(c) Yourself sprinting.

## 3 Manipulating Sinusoidal Functions

The general solution to the differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x(t) = 0$$

is

$$x_1(t) = C_1 \sin(\omega t) + C_2 \cos(\omega t)$$

For convenience, we often write this solution in the equivalent forms:

$$\begin{aligned} x_2(t) &= A \sin(\omega t + \phi) \\ x_3(t) &= B \cos(\omega t + \theta) \end{aligned}$$

Show that $x_1(t)$, $x_2(t)$, and $x_3(t)$ are equivalent by deriving equations for $A$ and $\phi$ in terms of $C_1$ and $C_2$, and for $B$ and $\theta$ in terms of $C_1$ and $C_2$. You may find the following trigonometric identities useful:

$$\begin{aligned} \sin(\alpha + \beta) &= \sin\alpha \cos\beta + \cos\alpha \sin\beta \\ \cos(\alpha + \beta) &= \cos\alpha \cos\beta - \sin\alpha \sin\beta \end{aligned}$$

## 4 Trigonometric Identities and Complex Numbers

The Euler formula states that:

$$e^{i\theta} = \cos\theta + i \sin\theta$$

Starting from this formula, show that:

(a) The exponential representation of the sine and cosine functions is:

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \qquad \sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$

(b) $\cos^2\theta + \sin^2\theta = 1$

(c) $\dfrac{d(\cos\theta)}{d\theta} = -\sin\theta$

(d) $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

## 5 Traveling Waves and Standing Waves

(a)

$$\Psi(x,t) = A\sin(kx - \omega t)$$

is a wave of wavelength $\lambda = 2\pi/k$ and frequency $\nu = \omega/2\pi$ traveling to the right. Derive an expression for the wave's velocity in terms of $\lambda$ and $\nu$.

(b) Write down the wavefunction for a wave traveling in the opposite direction.

(c) Show that when summed together with equal amplitudes and wavelengths, the counterpropagating waves from the first two parts of this question form a standing wave. Recall that a standing wave has a wavefunction which factors into two functions which depend purely on $x$ or $t$. You may want to use the trigonometric identities given in Problem 3 here.

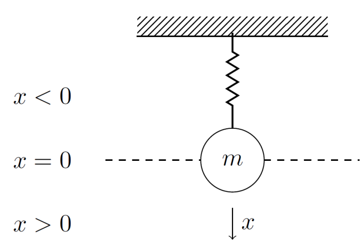
(d) Does the wavefunction

$$\Psi(x,t) = A\sin(kx - \omega t) + 2A\sin(kx + \omega t)$$

generate a traveling wave or a standing wave?

## 6 Classical Harmonic Oscillator:

To get some more practice with differential equations, let's consider the problem of the classical harmonic oscillator. Imagine a mass $m$ attached to a spring:




Suppose there is no gravitational force acting on $m$, so the only force is from the spring. Let the relaxed equilibrium position of the mass be $x = 0$. Hooke's law says that the force acting on the mass is $F = -kx$, where $k$ is the force constant of the spring, which describes its stiffness.
The momentum of the mass is:

$$p = m\frac{dx}{dt}$$

and Newton's second law says that the rate of change of momentum is equal to a force:

$$\frac{dp}{dt} = F$$

(a) Replacing $F$ by Hooke's law in the expression above, show that

$$m\frac{d^2x}{dt^2} = -kx$$

(b) Given that the mass starts at $x = 0$ with initial velocity $v_0$ at $t = 0$, show that its displacement is given by

$$x(t) = v_0 \left(\frac{m}{k}\right)^{1/2} \sin\left[\left(\frac{k}{m}\right)^{1/2} t\right]$$

(c) Describe the behavior of this solution. What does the motion look like? What is the amplitude of motion? What is the period of oscillation; that is, how long does it take for the system to undergo one complete cycle?

# 7 Damped Harmonic Oscillator

Let's modify the problem above to consider a damped harmonic oscillator, where the mass moves through a viscous medium that applies an opposing force proportional to the mass's velocity. This system has an equation of motion given by

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

where $\gamma$ is the viscous drag coefficient.

(a) Start by making the guess that $x(t) = A \cdot e^{\lambda t}$ is a valid solution to this differential equation. Write down an expression for $\lambda$ in terms of $m$, $\gamma$ and $k$.

(b) Sketch the behavior of $x(t)$ when $\gamma^2 \ll mk$.