Useful Integrals and Identities
CHM 305, Fall 2023

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1 \tag{1}$$

$$\frac{d(\sin\theta)}{d\theta} = \cos\theta \tag{2}$$

$$\frac{d(\cos\theta)}{d\theta} = -\sin\theta \tag{3}$$

$$\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right) \tag{4}$$

$$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right) \tag{5}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{6}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \tag{7}$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta) \tag{8}$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta) \tag{9}$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta) \tag{10}$$

$$\sin(\alpha)\cos(\alpha) = \frac{1}{2}\sin(2\alpha) \tag{11}$$

Trigonometric Integrals

$$\int \sin(\alpha x)dx = -\frac{1}{\alpha}\cos(\alpha x) \tag{12}$$

$$\int \sin^2(\alpha x)dx = \frac{x}{2} - \frac{\sin(2\alpha x)}{4\alpha} \tag{13}$$

$$\int x\sin^2(\alpha x)dx = \frac{x^2}{4} - \frac{x\sin(2\alpha x)}{4\alpha} - \frac{\cos(2\alpha x)}{8\alpha^2} \tag{14}$$

$$\int x^2\sin^2(\alpha x)dx = \frac{x^3}{6} - \left(\frac{x^2}{4\alpha} - \frac{1}{8\alpha^3}\right)\sin(2\alpha x) - \frac{x\cos(2\alpha x)}{4\alpha^2} \tag{15}$$

Gaussian Integrals

$$\int_0^\infty e^{-\alpha x^2} dx = \left(\frac{\pi}{4\alpha}\right)^{1/2} \tag{16}$$

$$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2a} \tag{17}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4a}\left(\frac{\pi}{a}\right)^{1/2} \tag{18}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1}\alpha^n}\left(\frac{\pi}{\alpha}\right)^{1/2} \tag{19}$$

$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}} \tag{20}$$